# Mortality Improvement: The Most Efficient Country

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The change in life expectancy over time can be expressed as

$$\dot{e}^o(t) = \mathrm{d}e^o(t)/\mathrm{d}t = \int_0^\omega \rho(x,t)e(x,t)f(x,t)\mathrm{d}x, \quad (1)$$

where $\rho(x,t) = -\frac{\partial \ln \mu(x,t)}{\partial t}$ is the rate of reducing mortality, $e(x,t) = \frac{\int_x^\omega \ell(a,t)\mathrm{d}a}{\ell(x,t)}$ the remaining life expectancy at age $x$, $f(x,t) = \ell(x,t)\mu(x,t)$ the death distribution of life table population, $\mu(x,t)$ the mortality at age $x$, and $\ell(x,t) = \exp(-\int_0^x \mu(a,t)\mathrm{d}a)$ the survival function at age $a$ (Vaupel and Canudas-Romo 2003).

Based on equation (1), Oeppen (2008) proposed a concept of efficiency of the age pattern of mortality change. Let

$$\varepsilon(x,t) = e(x,t)f(x,t). \quad (2)$$

Then, an "efficient" age pattern of mortality reduction should be the one that produces greater mortality reductions at ages where $\varepsilon(x,t)$ fractions are greater. Accordingly, France, Japan and Switzerland have been among the most efficient in recent years. The sum of (2) over age $x$ gives the number of remaining life expectancy lost due to death,

$$e^\dagger = \int_0^\omega e(x)f(x)\mathrm{d}x,$$

which can be used to measure lifespan disparity (Vaupel and Zhang 2006). If $e^\dagger$ is small, then most people die at roughly the same age, and vice versa.

The history of human mortality so far has shown a negative relationship between life expectancy and lifespan disparity. In particular, best practice countries have enjoyed not only the longest life years but the lowest or very low lifespan disparity as well. Resulting from the common progress of mortality improvement, life expectancy and lifespan disparity are just two sides of a coin. Thus, it is of interest to ask whether and how closely the efficiency in increasing life expectancy is related to that in decreasing lifespan disparity.

Before we start to investigate the efficiency, it is necessary to clarify the different impacts of mortality reductions on $e^o$ and $e^\dagger$. Any mortality improvement can increase life expectancy, no matter how such an improvement is distributed. Hence, the effect of mortality reduction on lifespan gains is consistent: the lower mortality, the higher life expectancy, or the inverse. On the other side, mortality improvement may either increase or decrease $e^\dagger$, depending on the age distribution of the improvement. It turns out that there exists a threshold age, denoted by $a^\dagger$, such that mortality reductions before it decrease $e^\dagger$, while those after it increase $e^\dagger$. These two different forces are defined as compression of mortality at younger ages and expansion of mortality at older ages , respectively. the change in $e^\dagger$ is the result of balance of the two forces (Zhang and Vaupel 2008).

Then lifespan disparity in the whole population is decomposed into two components according to $a^\dagger$

$$\begin{aligned} e^\dagger(t) &= \int_0^{a^\dagger} e(x,t)f(x,t)\mathrm{d}x + \int_{a^\dagger}^{\omega} e(x,t)f(x,t)\mathrm{d}x \\ &= e_c^\dagger(t) + e_e^\dagger(t). \end{aligned} \tag{3}$$

Since the driving force underlying equal life chances is compression at younger ages, it is $e_c^\dagger$ rather than $e^\dagger$ that can exactly capture the effect of mortality changes in equalizing life chances. $e_c^\dagger$ thus is better than $e^\dagger$ as a measure of lifespan disparity. Accordingly, the change in $e^\dagger$ over time can be decomposed into two components

$$\dot{e}^\dagger(t) = \int_0^{a^\dagger} \rho(x,t)k(x,t)f(x,t)\mathrm{d}x + \int_{a^\dagger}^{\omega} \rho(x,t)k(x,t)f(x,t)\mathrm{d}x, \tag{4}$$

where $k(x,t) = e^\dagger(x,t) - e(x,t)(1+\ln \ell(x,t))$ and $e^\dagger(x,t) = \int_x^\omega e(a,t)f(a,t)\mathrm{d}a/\ell(x,t)$. Let

$$\phi(x,t) = k(x,t)f(x,t) \tag{5}$$

indicate the efficient function of mortality changes on lifespan disparity. Then

$$\dot{e}^\dagger(t) = \int_0^{a^\dagger} \rho(x,t)\phi(x,t)\mathrm{d}x + \int_{a^\dagger}^{\omega} \rho(x,t)\phi(x,t)\mathrm{d}x. \tag{6}$$

Likewise, the change in $e^o$ as defined in (1) can be reexpressed based on threshold age

$$\begin{aligned} \dot{e}^o(t) &= \int_0^{a^\dagger} \rho(x,t)e(x,t)f(x,t)\mathrm{d}x + \int_{a^\dagger}^{\omega} \rho(x,t)e(x,t)f(x,t)\mathrm{d}x \\ &= \int_0^{a^\dagger} \rho(x,t)\varepsilon(x,t)\mathrm{d}x + \int_{a^\dagger}^{\omega} \rho(x,t)\varepsilon(x,t)\mathrm{d}x. \end{aligned} \tag{7}$$

Because the mortality improvement is common in the first component in both (6) and (7), the distance between $\varepsilon(x,t)$ and $|\phi(x,t)|$[1] can reflect the efficiency difference at age $x$ regarding increasing $e^o$ and decreasing $e_c^\dagger$, that is,

$$\delta(x,t) = \varepsilon(x,t) - |\phi(x,t)| \qquad \text{for } x \le a^\dagger. \tag{8}$$

[1]Note that $\phi(x,t)$<0 for $x < a^\dagger$ measures the efficiency of mortality improvement in *decreasing* lifespan disparity. Hence, $|\phi(x,t)|$ instead of a negative $\phi(x,t)$ should be used to compare with $\varepsilon(x,t)$.

Because $\varepsilon(x,t) > |\phi(x,t)|$ for $x \leq a^\dagger$, [2] the area between two them are the overall efficiency gap as below

$$\delta(t) = \int_0^{a^\dagger} \delta(x,t)\mathrm{d}x. \tag{9}$$

Given the same progress of mortality improvement, $e^o$ will be increased $\delta(t)$ years more than $e^\dagger$ will be decreased. A big $\delta$ implies that lifespan gains are achieved more than the gains against $e^\dagger$. With a small $\delta$, the mortality improvement can equalize life chances of the population almost as efficiently as it increases life expectancy.

Figure 1 depicts the two efficiency functions and the gap between them. It is obvious that the age distribution of efficiency gap changed a lot during the past 150 years. Before the 1950s, the major of efficiency gap was concentrated around infant/child mortality; and since then, the gap among adults became relatively big.

Figure 2 presents the efficiency gap in selected countries as well as the best practice country. Note that threshold age increases over time (see Figure 1 or Zhang and Vaupel 2008) so that the interval on which $\delta(x,t)$ is integrated becomes large. Even though, the sum of $\delta(x,t)$ on $[0,a^\dagger]$ has decreased over the past 150 years. This tells us that a big progress has been made for equalizing life chances, because the efficiency in decreasing lifspan disparity has been getting close to the efficiency in lifespan gains.

It is remarkable that best practice countries have kept the lowest $\delta(t)$ nearly throughout the whole past 150 years. In other words, best practice countries have done very well not only on the progress of reducing mortality, but also on equally distributing the benefits from mortality improvement over their members.

At the opposite extreme to the best practice country, the efficiency gap in the US has remained at a very high level since the mid-1950s, making up the upper boundary of $\delta(t)$ among selected countries. This is largely due to the US poor performs on equalizing life chances, as reported in many other studies (e.g., Murray et al. 2006). Besides, Japan experienced dramatical drop in the efficiency gap during thirty years after the World War II, and has held the lowest $\delta$ then. Many other countries lie between the best practice country and the US.

The concomitant of a high life expectancy and low lifespan disparity has been documented well in previous studies (e.g., Wilmoth and Horiuchi 1999; Vaupel and Zhang 2006). Taking a further step, the present study reveals that underlying such a concomitant is the resemblance of efficiency in terms of gains in lifespan and against lifespan disparity. If a country is efficient in increasing life expectancy, it is usually efficient in equalizing life chances too, and thus has a relatively high life expectancy and low lifespan disparity. Therefore, best practice countries with the highest life expectancy in the world is the most efficient.

[2] Note that

$$\begin{aligned} |\phi(x,t)| &= -\phi(x,t) \qquad \text{for } x \leq a^\dagger \\ &= -f(x,t)(e^\dagger(x,t) - e(x,t)(1-H(x,t))) \\ &= f(x,t)e(x,t) - f(x,t)(e^\dagger(x,t) + e(x,t)H(x,t)) \\ &= \varepsilon(x,t) - f(x,t)(e^\dagger(x,t) + e(x,t)H(x,t)). \end{aligned}$$

Then

$$\delta(x,t) = \varepsilon(x,t) - |\phi(x,t)| = f(x,t)(e^\dagger(x,t) + e(x,t)H(x,t)) > 0.$$

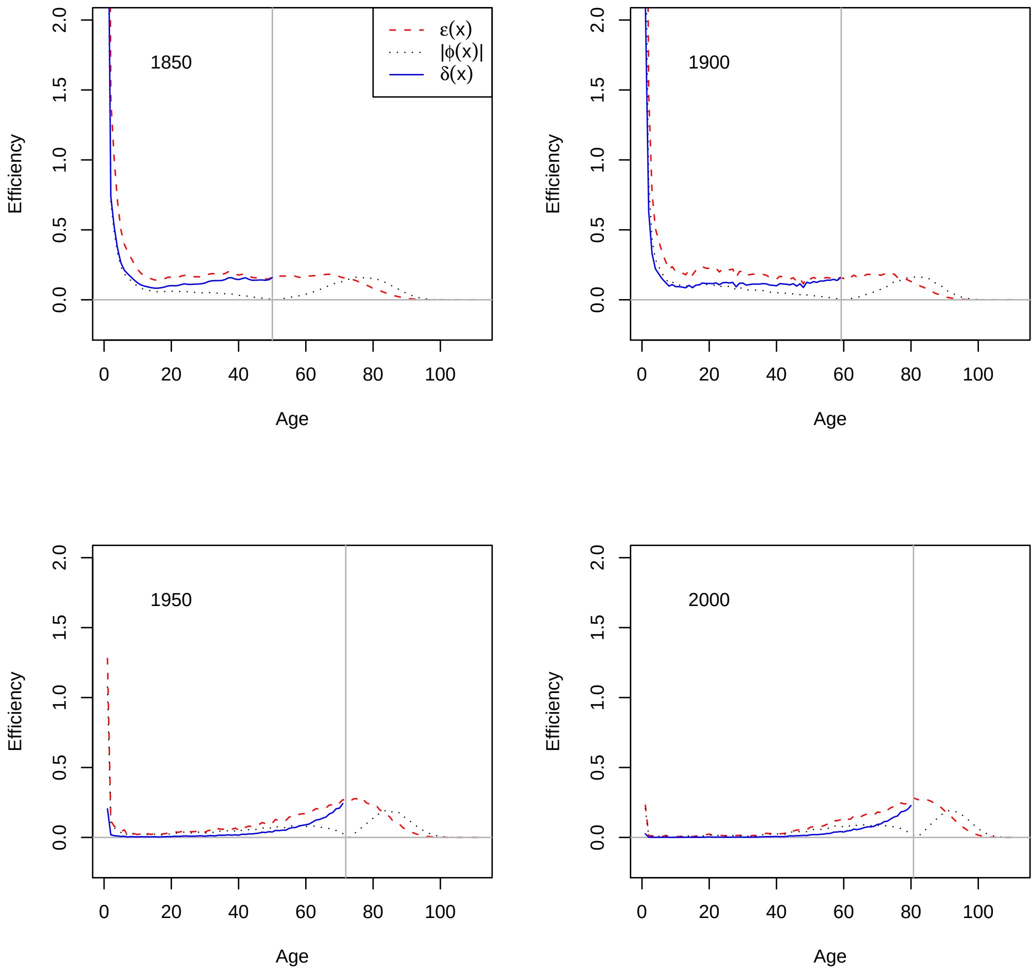


Figure 1: Efficiency of mortality reductions on changes in $e^o$ and $e^\dagger$ for Swedish females in 1850, 1900, 1950, and 2000. The dash lines stand for the efficiency function about increase in $e^o$ and the dot lines for that about decrease in $e^\dagger$, while the solid lines for efficiency gap between the two functions. Source: Human Mortality Database 2008.

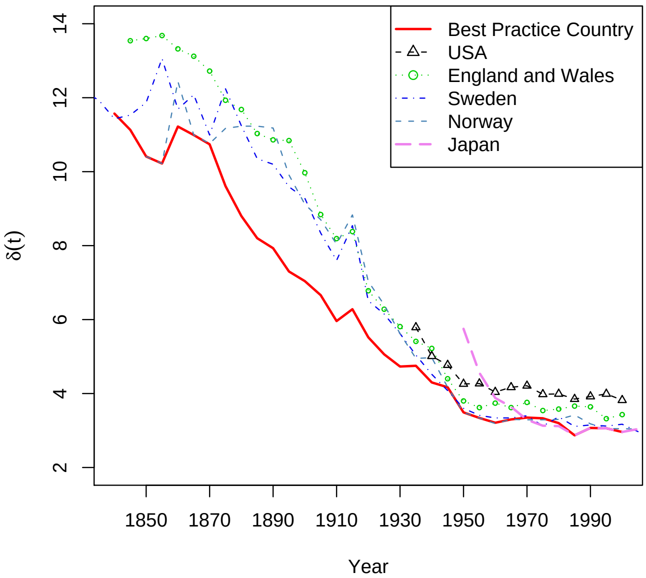


Figure 2: Distance between two efficiency functions of mortality changes in increasing $e^o$ and decreasing $e^\dagger$, females, selected country vs. best practice country. Source: Human Mortality Database 2008

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