# Subcounty Population Forecast Accuracy: 

## An Analysis Beyond Size and Growth

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Population forecasts are produced at many levels of geography. In the United States, the U.S. Census Bureau constructs forecasts for the nation and all states. State forecasts are also produced by many members of the Federal-State Cooperative Program for Population Projections (FSCPP) and FSCPP affiliates are the primary producers of county forecasts as well. A few states such as Arizona, Massachusetts, and Wisconsin create forecasts for municipalities, but forecasts for subcounty areas such as cities, census tracts, and zip codes are more commonly produced by local governments and planning agencies. A number of business and non-profit groups also produce population forecasts for geographic areas extending from nations down to small subcounty areas.

Population forecasts play a critical role in many types of planning, budgeting, and policy decisions; consequently, forecast accuracy is of great concern to analysts and decision-makers in both the public and private sectors. Forecast accuracy has been evaluated rather extensively for nations, states, and counties (e.g., Campbell, 2002; Keilman, 1997; Morgenroth, 2002; Rayer, 2007; Smith, 1987; Smith and Sincich, 1988, 1992; White, 1954) but studies at the subcounty level have been far less common and have often been limited in their spatial and temporal scope. Perhaps the most extensive investigation of forecast accuracy at the subcounty level was conducted by Isserman (1977), whose study covered forecasts for 1960 and 1970 for 1,777 townships in Illinois and Indiana. Also, Murdock et al. (1984) examined forecasts for 553 incorporated places in Texas and North Dakota for 1980; Smith and Shahidullah (1995) analyzed accuracy for census tracts for three counties in Florida for 1990; Tayman (1996) evaluated forecasts for census tracts in San Diego County for 1990; and Tayman, Schafer, and

Carter (1998) investigated forecast accuracy for randomly selected grid cells in San Diego County for 1990.

Most research at the subcounty level has focused on the accuracy of forecasts of total population, sometimes investigating the effects of differences in population size and growth rate as well. In the present study, we evaluate the accuracy of forecasts for incorporated places and unincorporated areas in Florida using data from 1970 to 2005. We consider the effects of differences in population size and growth rate, but extend the analysis to account for changes in special populations and annexations. To our knowledge, the impact of these factors on forecast accuracy has not previously been studied. Although special populations such as prisoners and college students can affect forecasts of larger areas such as states and counties, they are of special concern at the subcounty level because they often account for a much larger proportion of total population. The same is true for annexations, which occur almost exclusively at the subcounty level. We also analyze several averaging techniques to determine whether they can improve the performance of the individual forecasting techniques. We conclude by summarizing our findings and making several recommendations regarding the production of subcounty population forecasts.

## DATA AND TECHNIQUES

This study analyzes forecast errors at the subcounty level for Florida for the period 1970 to 2005 . The population data for $1970,1980,1990$, and 2000 are decennial census counts from the U.S. Census Bureau. We prepared a series of mid-decade estimates for 1975, 1985, and 1995 using residential electric customer data, decennial census counts, and
interpolated population/electric customer ratios. These estimates were adjusted in some areas to account for apparent data problems. Estimates for 2005 were produced by the Bureau of Economic and Business Research (BEBR) at the University of Florida (Bureau of Economic and Business Research, 2006).

The subcounty areas used in the study cover the entire territory of each county and consist of incorporated places and unincorporated areas. The former include cities, towns, and villages; the latter make up the remainder of each county. Only places that have been incorporated throughout the entire study period are included in the analysis, resulting in a sample of 383 incorporated places. Places that incorporated after 1970 were assigned to the unincorporated area of their respective counties. There are 66 unincorporated areas in the analysis, one for each county in Florida with the exception of Duval County, whose entire territory is incorporated.

Following Smith, Tayman, and Swanson (2001), we use the following terminology to describe population forecasts:

1) Base year: the year of the earliest population size used to make a forecast.
2) Launch year: the year of the latest population size used to make a forecast.
3) Target year: the year for which population size is forecasted.
4) Base period: the interval between the base year and launch year.
5) Forecast horizon: the interval between the launch year and target year.

For example, if data from 1970 and 1980 were used to forecast population in 1990, then 1970 would be the base year, 1980 would be the launch year, 1990 would be the target year, 1970-1980 would be the base period, and 1980-1990 would be the forecast horizon.

Using data for the period 1970 to 2005, we constructed forecasts using base periods extending in 5-year intervals from five to 30 years and horizons extending in 5year intervals from five to 30 years. This led to forecasts for 56 different forecast horizon / base period / target year combinations, including 21 five-year forecasts, 15 ten-year forecasts, ten 15-year forecasts, six 20-year forecasts, three 25 - year forecasts, and one 30-year forecast. For each of these, we applied six commonly used techniques, including three extrapolation techniques and three ratio techniques. The former include linear (LIN), exponential (EXP), and constant (CON); the latter include share-of-growth (SHR), shift-share (SFT), and constant-share (COS). From these individual techniques we constructed two more forecasts, one an average of forecasts from all six techniques (AV) and one an average after the highest and lowest forecasts were excluded (TAV). We refer to the former as an overall average and the latter as a trimmed average. A mathematical description of the individual techniques is shown in the appendix.

We examine forecast accuracy in two ways, one reflecting precision and the other bias. Precision refers to the difference between forecasts and subsequent census counts or population estimates, ignoring whether those forecasts were too high or too low; bias refers to the tendency for forecasts to be too high or too low by accounting for the direction of errors.

With regard to precision, the most commonly used error measure is the mean absolute percent error, or MAPE. It is calculated as follows:

$$
M A P E=\Sigma\left|P E_{t}\right| / n, \quad P E_{t}=\left[\left(F_{t}-A_{t}\right) / A_{t}\right] * 100
$$

where $P E$ represents the percent error, $t$ the target year, $F$ the population forecast, $A$ the actual population, and $n$ the number of areas. Forecasts that are perfectly precise result in
a MAPE of zero. The MAPE has no upper limit - the larger the MAPE, the lower the precision of the forecasts.

The mean algebraic percent error (MALPE) is often used as a measure of bias. It can be calculated analogously to the MAPE, using algebraic rather than absolute percent errors:

$$
M A L P E=\Sigma P E_{t} / n
$$

Negative values of the MALPE indicate a tendency for forecasts to be too low, whereas positive values indicate a tendency for them to be too high. Being arithmetic means, the MAPE and MALPE are susceptible to outliers, but both measures are commonly used and are generally sufficient to summarize the error distribution of population forecasts (Isserman 1977; Rayer 2007; Smith 1987; Tayman et al. 1998).

Choosing the appropriate base period is one of the first decisions that must be made when constructing population forecasts. A general recommendation is that the length of the base period should correspond to the length of the forecast horizon (Alho and Spencer 1997). However, studies investigating this issue have not provided strong support for this recommendation. Smith and Sincich (1990) found that the length of the base period had little impact on the precision of state population forecasts covering very short horizons, but that 10-year base periods were generally necessary (and sufficient) to ensure the greatest possible precision for horizons extending 10 years or more. Beaumont and Isserman (1987) reported that precision improved for a sample of rapidly growing states when the base period was extended from 10 to 40 years for forecasts made using the exponential technique. However, longer base periods did not improve precision for forecasts made using the linear technique. At the county level, Rayer (2008) found small
improvements in precision when the base period was extended from 10 to 20 years for 10-30 year forecast horizons. Improvements were fairly small for most techniques but were substantial for the exponential technique, especially for longer horizons.

Lengthening the base period beyond 20 years yielded no further improvements and actually lowered precision in some instances. None of these studies found a consistent relationship between the length of the base period and the tendency for forecasts to be too high or too low.

We found that increasing the length of the base period from five to 10 years generally improved the precision of population forecasts but that further increases had little additional impact (data not shown). Consequently, we use 10-year base periods for the remainder of our analysis. To keep the presentation of results succinct, in the following section we focus on forecasts with 10- and 20-year horizons.

## BASIC RESULTS

## Accuracy by Population Size

Previous research has found population size to affect the precision but not the bias of population forecasts (e.g., Rayer, 2008; Smith and Shahidullah, 1995; Smith and Sincich, 1998; Tayman et al., 1998). Forecasts generally become more precise as population size increases, at least up to some relatively large size. Consequently, on average, forecasts for the nation tend to be more precise than forecasts for states, forecasts for states tend to be more precise than forecasts for counties, and forecasts for counties tend to be more precise than forecasts for subcounty areas.

Table 1a shows MAPEs by population size in the launch year for the six individual forecasting techniques and two averages. As expected, the forecasts generally become more precise as population size increases. The largest improvements in precision occur primarily in the smallest size categories. MAPEs are very large for the smallest subcounty areas (especially for the 20-year horizon), but decline considerably as population size increases to 5,000 . Beyond that level, however, further increases in population size lead to relatively small additional declines.
(Table 1 about here)
For several techniques MAPEs increase as population size increases beyond a certain level, especially for longer forecast horizons. This apparent anomaly can be explained by the confounding influence of population growth. With few exceptions, subcounty areas in the two largest size categories had higher growth rates than those in the smaller size categories (data not shown). As discussed in the following section, high growth rates are generally associated with relatively large MAPEs; consequently, the elevated MAPEs shown in some of the larger size categories in Table 1a can be explained by the high rates of population growth. We consider the joint effects of population size and growth rates later in the paper.

Table 1 b shows conflicting results regarding the relationship between population size and bias. MALPEs sometimes decline as population size increases, sometimes increase, sometimes display a u-shaped relationship, and sometimes follow no clear pattern. We believe these inconsistent results are caused both by the lack of a strong relationship between bias and population size and by the confounding influence of
population size and growth rates on forecast errors. We return to this point later in the paper.

## Accuracy by Growth Rate

Previous research has found population growth to have a consistent impact on both precision and bias. In general, forecasts tend to be most precise for areas with slow but positive growth rates and least precise for areas experiencing large positive or negative growth rates (e.g., Keyfitz, 1981; Murdock et al., 1984; Smith and Sincich, 1992; Stoto, 1983; White, 1954). Furthermore, forecasts tend to be too high in areas that grew rapidly during the base period and too low in areas that declined or grew very slowly (e.g., Isserman, 1977; Rayer, 2008; Smith, 1987; Smith and Sincich, 1988; Tayman, 1996).

Table 2a shows the well-known u-shaped relationship between growth rates and precision. For all but the constant-share technique for 20-year horizons, MAPEs are highest for areas that either grew or declined rapidly during the base period and lowest for areas with slow to moderate growth rates. Error levels themselves differ substantially from one forecasting technique to another. For areas with declining populations, the constant and exponential techniques generally provide the most precise forecasts, and shift-share the least precise. For areas that grew particularly rapidly, the linear technique has the smallest errors and the exponential technique the largest. Constant-share, while associated with relatively low precision overall, is almost as precise as the linear technique for areas experiencing high rates of population growth. We return to these findings later in the paper when we discuss composite forecasts.
(Table 2 about here)

With respect to bias, Table $2 b$ confirms findings reported in several previous studies that there is a strong tendency for forecasts to be too low in areas that declined during the base period and too high in areas that grew rapidly. This is true for all but the constant-share and constant techniques. The former exhibits a positive bias that declines as the growth rate increases while the latter exhibits a negative bias that becomes greater. In general, MALPEs follow a stepwise pattern for most techniques: with increasing rates of population growth, MALPEs become less negative or more positive (again, the constant-share and constant techniques are exceptions). Extending the forecast horizon from 10 to 20 years accentuates this pattern.

## Accuracy by Size and Growth

The preceding discussion touched on the interrelationship between population size and rate of growth. To disentangle the effects of these two variables, we evaluate forecast errors for combined size and growth categories. For most techniques, precision increases with increasing population size within each growth category (see Table 3a). Within each size category, MAPEs are highest for areas with either declining or rapidly growing populations and lowest for areas with moderate growth rates. Both results confirm findings reported previously.
(Table 3 about here)
With respect to bias, the data in Table 3 b show two separate results. Within each size category, there is a strong positive relationship between MALPEs and population growth for all techniques except constant and constant-share: errors are large and negative for areas with negative growth rates and become positive and larger as the
growth rates increase. These results are consistent with those shown in Table 2b. Within growth rate categories, however, there is no clear relationship between MALPEs and population size. In some instances MALPEs decline as population size increases, in other instances they increase. These results support the conclusion that population size is not closely related to forecast bias.

## Combining Forecasts

Researchers in various disciplines have advocated the production of forecasts based on combining the results of several individual forecasting techniques (e.g., Armstrong, 2001; Clemen, 1989; Makridakis et al. 1998; Webby and O’Connor, 1996). In population forecasting, these "combined" forecasts have often been found to be more accurate than the individual forecasts used in their construction (Ahlburg, 1995; Isserman, 1977; Rayer, 2008; Smith and Shahidullah, 1995). Overall averages or trimmed averages have been the most common techniques used in combining forecasts, but other approaches can be applied as well.

In this study, we developed the two averages described above. They generally performed very well, but the performance of the overall average was strongly affected by the large errors associated with the exponential technique for longer forecast horizons. This suggests that it may not be advisable to rely on an overall average because outliers associated with a single technique can greatly affect the results. The trimmed average generally fared better than the overall average, but in many instances the trimmed average was not quite as accurate as the most accurate individual technique.

The results summarized in Tables 1 through 3 showed that some techniques performed better than others for areas with particular size or growth rate characteristics. This information can be used to develop composite forecasts based on specific combinations of individual techniques (e.g., Isserman, 1977). We develop three alternative composite techniques, one based on performance by population size alone (C1), one based on performance by growth rate alone (C2), and one based on the combined performance by population size and rate of growth (C3). Composite C 1 selects the constant technique for areas with a launch year population of 2,000 or less and the linear technique otherwise. Composite C 2 selects the constant technique for areas that lost population over the base period and the linear technique otherwise. Composite C3 selects the constant technique for areas that lost population over the base period irrespective of population size; the constant technique for areas with a population of 2,000 or less irrespective of the rate of growth; and the linear technique for areas that grew over the base period and had a launch year population exceeding 2,000 .

Table 4a show MAPEs by forecast horizon for the six individual techniques, the overall average, the trimmed average, and the three composite techniques. All three composite forecasts are more precise than any of the individual or averaged techniques for all forecast horizons. Of the three composites, C3 performs best and C2 worst, although C1 is almost as precise as C3. With respect to bias (Table 4b), all three composites have less bias than the other techniques in most instances. C 1 and C 3 once again show similar results with C3 having slightly less bias. Accounting for both population size and growth rate thus led to better results than accounting for either characteristic by itself.
(Table 4 about here)
The composite approach clearly performed well for the subcounty population forecasts analyzed in this study. For general forecasting purposes, however, the usefulness of the composite approach depends on whether similar results would be found for other time periods and geographic areas. To examine this issue, we applied the same forecasting techniques used in the preceding analysis to a large sample of counties in the continental United States and decennial census data from 1900 to 2000.

We found that the relative performance of the individual techniques by population size and growth rate was about the same for the national sample of county forecasts as for subcounty forecasts in Florida (data not shown). The constant technique was the most precise for counties with a population of 10,000 or less; for larger counties the linear technique performed best. Linear was the best performing technique for counties with positive growth over the base period; for counties losing population, the exponential technique performed best, followed closely by the constant technique. When population size and growth rate were combined, the results resembled those of the subcounty analysis. From these results we again developed three composites (C1-C3). The individual techniques used in these composites are listed at the bottom of Table 5b.
(Table 5 about here)
Table 5a shows that with respect to precision the differences among the various techniques are smaller at the county than at the subcounty level. The trimmed average performs well throughout, showing the lowest MAPEs of any technique for short horizons and only marginally trailing the three composites for longer horizons. Of the three composites C3 is again the most precise technique overall, but not by much. The
trimmed average also excels with respect to bias. The three composites show slightly higher levels of bias than the trimmed average but outperform most individual techniques.

To summarize, in both data sets a combination of forecasts based on averages and composite techniques generally provided more precise and less biased forecasts than any of the individual techniques. While there were some differences in results between the two data sets, the similarities are remarkable, especially given the range of geographic areas and time periods covered. The composite techniques generally outperformed both averages, although the differences were very small in the county data set. We also evaluated several more complex composite models that included averages of several techniques for each size and growth category, but were unable to consistently improve on the results presented here (data not shown). We believe combinations of forecasts especially trimmed averages and composite techniques - will generally produce more accurate small-area forecasts than can be obtained by the application of any single technique. Further research may uncover new and better techniques for combining forecasts than those presented here.

## EXTENDING THE ANALYSIS

The analysis thus far has focused on the effects of differences in population size and growth rate on forecast accuracy and the potential benefits of combining forecasts. We turn now to two additional factors that may play an important role at the subcounty level: special populations and annexations. To our knowledge, the effect of these two factors on population forecast accuracy has not been previously studied.

## Accounting for Special Populations

A special population can be defined as "a group of persons that is found in a locality usually by reason of an administrative decision or legislative fiat" (Pittenger, 1976: 205). These include groups such as college students, inmates of correctional facilities, and residents of military barracks and nursing homes. Special populations can present a challenge to population forecasters because they often have unique demographic characteristics and may follow different growth trajectories than the rest of the population. For example, college students are heavily concentrated in the 18-24 age group and generally maintain the same age profile over time, and the number of prison inmates in a particular locality can increase or decline regardless of overall population growth trends. If special populations are not explicitly accounted for in the forecasting process, they may lead to unrealistic patterns of population change.

In general, adjustments for special populations are only needed when these groups comprise a substantial proportion of the total population and if their growth patterns differ from those of the rest of the population. Unfortunately, there are no general guidelines that define how 'different' and 'substantial' a special population has to be to cause problems in population forecasting, and it is up to the analyst to make that assessment (Smith et al. 2001).

A common technique for adjusting for special populations is to subtract them from the base-period data, make a forecast of the remaining population, and add back an independent forecast of the special population in the target year (Smith et al. 2001). We use this technique to investigate whether accounting separately for special populations
improves forecast accuracy. The special populations we consider are inmates and patients in institutions operated by the federal government, the Florida Department of Corrections, and the Florida Department of Children and Family Services.

We follow two different approaches when adding back a forecast of the special population in the target year: 1) We hold the special population constant at its launch year value (SP1), and 2) We add its actual target year value (SP2). The former reflects a naïve assumption of no change in special populations and can be thought of as a worst-case scenario; the latter is an ideal-case scenario, showing the maximum potential improvement that could be achieved with a perfect forecast of the special population.

There were 141 subcounty areas that had special populations during the study period, accounting for slightly less than one-third of the total number of areas. We made three forecasts for each area: one with no adjustment for special populations, one using the SP1 adjustment, and one using the SP2 adjustment. Table 6 summarizes the impact of these adjustments on forecast precision, showing the change in MAPEs produced by each of the two adjustment techniques. A negative sign indicates that the adjustment reduced the MAPE and a positive sign indicates the opposite. We present the results for all horizon lengths and all target years for all 141 subcounty areas with special populations and for areas in which special populations exceeded $1 \%, 2.5 \%$, and $5 \%$ of the total population. We limit the analysis to forecast precision because adjusting for special populations was found to have no consistent effect on bias.
(Table 6 about here)
The results shown in Table 6 are based on forecasts from the linear technique. We focus on the linear technique because using the trimmed average is problematic for this
analysis. The trimmed average includes as many as three ratio techniques. As described in the appendix, in the ratio techniques the population (or population change) of a smaller area is expressed as a proportion of population (or population change) of a larger area. In the previous sections of the paper, the county represented the larger area. Here, we are dealing with adjustments for subsets of areas, making the choice of the larger area somewhat arbitrary. We favor the linear technique over the exponential and constant techniques because the exponential technique is prone to extreme forecasts and the constant technique exhibits a strong negative bias. Furthermore, in the overall analysis, results for the trimmed average are closer to the results for the linear technique than to any of the other individual techniques.

Table 6 shows that adjusting for special populations leads to relatively small but consistent improvements in precision when there are perfectly accurate forecasts of special populations (SP2). Holding special populations constant over the forecast horizon (SP1) provides mixed results; for some target years and horizons, this adjustment improves precision, for others it reduces precision. For SP2, the improvements achieved by adjusting for special populations become consistently larger as special populations increase as a percentage of total population. Results for SP1 are not as consistent in this regard.

What do these results imply for the producers of small-area population forecasts? In the present data set, only with perfect information does accounting for special populations provide a consistent benefit; holding special populations constant does not always improve precision and in many cases actually reduces it. Whether the potential gains from accounting for special populations is worth the additional effort can be
debated, especially since it appears that reasonably accurate forecasts of the growth in the special population are needed to improve overall forecast accuracy. However, it should be noted that improvements in precision become larger as special populations account for a greater percentage of the total population. It might also be mentioned that accounting explicitly for special populations may be politically advantageous by showing that the analyst is trying to account for relevant factors. Consequently, in areas where special populations exceed a small proportion of the total, and where they exhibit different growth patterns than the rest of the population, we believe it is generally advisable to account for those populations separately when preparing population forecasts.

## Accounting for Annexations

Annexations can also pose a challenge when making forecasts for small areas because they introduce changes in geographic boundaries into the forecasting process. Although annexations are rare at the state and county level, in many states - including Florida they are a common occurrence at the subcounty level and often have significant demographic consequences (Raymondo, 1992, p. 77). Some incorporated places annex adjoining areas on a regular basis, but in most instances annexations occur infrequently and irregularly. If the demographic effects of annexations are not accounted for explicitly, the analyst is essentially forecasting that similar effects will continue in the future. Consequently, it may be reasonable to treat the annexed population in a manner similar to that used for special populations.

In order to evaluate the effect of adjusting for annexations, we compare unadjusted forecasts with forecasts in which the population annexed during the base
period is subtracted from the total population in the launch year, a forecast of the nonannexed population is made using the techniques described previously, and the annexed population in the target year is added to the resulting forecast. Once again, we present results for two different approaches: under the first, we assume that no further annexations occur (A1) and under the second, we add the population effects of annexations that actually occurred during the forecast horizon (A2). The first approach reflects the naïve assumption of no change in annexed populations; as was true for special populations, it can be thought of as a worst-case scenario. The second approach is an ideal-case scenario, showing the maximum potential improvement that could be achieved with a perfect forecast of annexed population. Again, we report the results only for the linear technique and differentiate among all subcounty areas that experienced annexations and those with annexations greater than $1 \%, 2.5 \%$, and $5 \%$ of the total population.

Evaluating the impact of annexations involves one complication that did not arise in the analysis of special populations. Annexations typically involve the expansion of a city's or town's boundary to encompass a previously unincorporated geographic area; consequently, cities and towns generally experience a population increase and unincorporated areas generally experience a population decline. We therefore investigate the impact of annexations separately for incorporated places and unincorporated areas. Tables 7 and 8 are structured analogously to Table 6 , with Table 7 showing results for incorporated places and Table 8 for unincorporated areas.
(Tables 7 and 8 about here)
Adjusting for annexations improves precision for the 183 incorporated places in this subsample (see Table 7). The improvements become larger as the forecast horizon
becomes longer and as the relative size of the annexation increases. In contrast to accounting for special populations, where only the scenario with perfect information (SP2) improved precision consistently, both annexation adjustment techniques lead to improvements in precision, though the effects are stronger for A2 than for A1.

Table 8 shows corresponding results for unincorporated areas. The results more closely resemble those for special populations than those for annexations into incorporated places. That is, having perfect information about future annexations leads to fairly consistent improvements in precision but assuming that no further annexations will occur provides mixed results, sometimes raising errors and other times reducing them. Once again, the effects of accounting for annexations become stronger with increases in the relative size of the annexation.

What accounts for the differences in results between incorporated places and unincorporated areas? We believe the explanation lies in the frequency of annexations in these two types of areas. Almost $30 \%$ of incorporated places with annexations had an annexation in only one of the seven 5 -year periods covered by the study and $12 \%$ had annexations in all seven periods. In contrast, for unincorporated areas with annexations, fewer than $18 \%$ had annexations in only one period and $33 \%$ had annexations in all seven periods (data not shown). Clearly, annexations are much more frequent in unincorporated areas than in incorporated places. Consequently, the naïve assumption (A1) improves precision in incorporated places but not in unincorporated areas.

We believe it is generally advisable to adjust for annexations when making smallarea forecasts, at least for areas in which annexations occur infrequently and account for more than a trivial proportion of total population. When areas have a history of frequent
annexations, however, such adjustments are not likely to lead to much improvement in forecast accuracy and may even make it worse. Thus, the analyst once again has to weigh the relatively small gains in precision and the potential political advantages of making adjustments against the costs of collecting additional data and amending the forecasting methodology.

## SUMMARY AND CONCLUSIONS

What can we say about precision and bias that might help practitioners as they construct subcounty population forecasts? Based on this study and the results of previous research, we have drawn the following conclusions:

1) For simple extrapolation and ratio techniques such as those evaluated in this paper, 10 years of base data are generally necessary to achieve the greatest possible forecast accuracy. In most instances, 10 years are also sufficient, as increases beyond 10 years generally lead to little if any further improvement in accuracy.
2) Precision declines steadily with the length of the forecast horizon, but bias follows no clear pattern. We found MAPEs to grow about linearly with increases in the forecast horizon, but MALPEs sometimes increased and other times declined. We also found that forecast errors for subcounty areas are often very large, especially for areas with small populations, high positive or negative growth rates, and long forecast horizons.
3) Forecast precision is positively related to population size, but bias is not. For every technique, MAPEs were larger for subcounty areas with fewer than 500 residents than for areas in any other size category, often by a substantial amount. For most techniques, MAPEs declined fairly steadily as population size increased to around 5,000 , but beyond
that did not change substantially. Except for the constant and constant-share techniques, MALPEs did not exhibit any clear relationship with population size; for those two techniques the relationship was negative, reflecting the generally positive correlation between population size and growth rates.
4) Population growth rates over the base period have a substantial impact on forecast accuracy. For all techniques except constant-share, MAPEs displayed a u-shaped relationship with the growth rate: Errors were smallest for areas with moderate growth rates and increased as growth rates deviated in either direction from those moderate levels. For all but the constant and constant-share techniques, MALPEs were large and negative for areas with the largest negative growth rates and increased as the growth rate increased, becoming large and positive for areas that grew rapidly during the base period. We believe these patterns characterize most forecasting techniques.
5) Taking averages of forecasts from several techniques often improves forecast accuracy. We found the trimmed average to produce errors that were smaller than the errors for most (sometimes all) of the individual techniques. However, we also found that a composite approach - using particular techniques or averages for areas with particular characteristics - worked even better. The performance of the composite approach was not limited to subcounty areas in Florida; when we extended the analysis to a large sample of counties in the United States using one hundred years of census data, we found similar results. Not only did the composites perform best in both data sets, the techniques used to construct them were similar as well. This demonstrates that knowledge of prior population size and growth rates can be used to construct forecasts that often outperform individual techniques and even trimmed averages. We believe the use of averaging and
the development of composite techniques hold a great deal of promise for small-area forecasting.
6) Accounting separately for changes in special populations may improve the average precision of population forecasts, but probably not by much. We found that adjusting for special populations reduced MAPEs consistently only when special populations were added using perfect information. Holding special populations constant did not yield appreciable benefits. We did not find a consistent impact on forecast bias under either scenario. Although the benefits of adjusting for special populations appear small, we believe such adjustments are generally advisable for public relations purposes and because they may have a significant impact on forecast accuracy in a few places. Further research is required before we can draw firm conclusions on this point.
7) Accounting for the effects of annexations appears to have a somewhat greater impact on forecast precision than accounting for changes in special populations, especially for incorporated places. We found that these improvements became greater as the forecast horizon became longer and as annexations became larger relative to the size of the entire population. Similar to the analysis of special populations, the approach using perfect information provided significantly better results than the approach assuming that no further annexations would occur. Unfortunately, forecasting the population effects of future annexations is more difficult than forecasting special populations. Nonetheless, we believe it is generally advisable to account for annexations when making subcounty population forecasts, at least when those annexations comprise a significant proportion of the total population.

## APPENDIX: FORECASTING TECHNIQUES

Extrapolation techniques express future population values solely as a function of past population values:

LIN: In the linear technique, the population increases (or declines) by the same number of persons in each future year as the average annual increase (or decline) observed during the base period:

$$
P_{t}=P_{l}+(x / y) *\left(P_{l}-P_{b}\right),
$$

where $P_{t}$ is the population in the target year, $P_{l}$ is the population in the launch year, $P_{b}$ is the population in the base year, $x$ is the number of years in the forecast horizon, and $y$ is the number of years in the base period.

EXP: In the exponential technique, the population grows (or declines) by the same rate in each future year as the average annual rate during the base period:

$$
P_{t}=P_{l} \mathrm{e}^{r x}, \quad r=\left[\ln \left(P_{l} / P_{b}\right)\right] / y,
$$

where e is the base of the natural logarithm and $\ln$ is the natural logarithm.
CON: In the constant technique, the population in the target year is the same as it was in the launch year:

$$
P_{t}=P_{l} .
$$

Ratio techniques express population (or population change) of a smaller area as a proportion of population (or population change) of a larger area in which the smaller area is located. In our analysis of subcounty areas, we use counties as the larger areas and produce county population forecasts by taking an average of forecasts from the linear and exponential techniques. In our analysis of counties, the population of the larger area is the sum of the populations of all the counties in the sample. In the following formulas,
subscripts denote subcounty-level values and superscripts denote county-level values for the subcounty level analysis; for the county level analysis the subscripts denote countylevel values and superscripts denote the sum total of all counties in the sample.

SHR: In the share-of-growth technique, a smaller area's share of population growth of the larger area is the same throughout the forecast horizon as it was during the base period:

$$
P_{t}=P_{l}+\left[\left(P_{l}-P_{b}\right) /\left(P^{l}-P^{b}\right)\right] *\left(P^{t}-P^{l}\right)
$$

SFT: In the shift-share technique, the annual change in a smaller area's share of population of the larger area is the same throughout the forecast horizon as it was during the base period:

$$
P_{t}=P^{t} *\left[P_{l} / P^{l}+(x / y) *\left(P_{l} / P^{l}-P_{b} / P^{b}\right)\right]
$$

COS: In the constant-share technique, a smaller area's share of population of the larger area in the target year is the same as it was in the launch year:

$$
P_{t}=\left(P_{l} / P^{l}\right) * P^{t}
$$

## REFERENCES

Ahlburg, D. A. 1995. Simple versus complex models: Evaluation, accuracy, and combining. Mathematical Population Studies 5: 281-290.

Alho, J., and B. Spencer. 1997. The practical specification of the expected error of population forecasts. Journal of Official Statistics 13: 203-225.

Armstrong, J. S. (2001). Combining forecasts. In J. S. Armstrong (Ed.), Principles of forecasting (pp. 417-439). Boston: Kluwer Academic Publishers.

Bureau of Economic and Business Research. 2006. Florida Estimates of Population: April 1, 2005. Gainesville: University of Florida.

Campbell, P.R. (2002). Evaluating forecast error in state population projections using Census 2000 counts, Population Division Working Paper Series No. 57. Washington, DC: US Census Bureau.

Clemen, R. (1989). Combining forecasts: A review and annotated bibliography. International Journal of Forecasting 5: 559-583.

Isserman, A. 1977. The accuracy of population projections for subcounty areas. Journal of the American Institute of Planners 43: 247-259.

Keilman, N. (1997). Ex-post errors in official population forecasts in industrialized countries. Journal of Official Statistics, 13, 245-277.

Keyfitz, N. (1981). The limits of population forecasting. Population and Development Review, 7, 579-593.

Makridakis, S., S. Wheelwright, and R. Hyndman. 1998. Forecasting: Methods and applications. New York: Wiley.

Morgenroth, E. (2002). Evaluating methods for short to medium term county population forecasting. Journal of the Statistical and Social Inquiry Society of Ireland, 31, 111-136.

Murdock, S., R. Hamm, P. Voss, D. Fannin, and B. Pecotte. 1991. Evaluating small-area population projections. Journal of the American Planning Association 57: 432443.

Murdock, S., L. Leistritz, R. Hamm, S.-S. Hwang, and B. Parpia. 1984. An assessment of the accuracy of a regional economic-demographic projection model. Demography 21: 383-404.

Pittenger, D. B. 1976. Projecting State and Local Populations. Cambridge, MA: Ballinger Publishing Company.

Rayer, S. 2007. Population forecast accuracy: Does the choice of summary measure of error matter? Population Research and Policy Review 26: 163-184.
__. 2008. Population forecast errors: A primer for planners. Journal of Planning Education and Research 27 (4): 417-430.

Raymondo, J. C. 1992. Population estimation and projection: Methods for marketing, demographic, and planning personnel. New York, NY: Quorum Books.

Smith, S. 1987. Tests of forecast accuracy and bias for county population projections. Journal of the American Statistical Association 82: 991-1003.

Smith, S., and M. Shahidullah. 1995. An evaluation of population projection errors for census tracts. Journal of the American Statistical Association 90: 64-71.

Smith, S., and T. Sincich. 1988. Stability over time in the distribution of population forecast errors. Demography 25: 461-474.
-_. 1992. Evaluating the forecast accuracy and bias of alternative population projections for states. International Journal of Forecasting 8: 495-508.

Smith, S., J. Tayman, and D. Swanson. 2001. State and local population projections: Methodology and analysis. New York, NY: Kluwer Academic/Plenum Publishers.

Stoto, M. A. (1983). The accuracy of population projections. Journal of the American Statistical Association, 78, 13-20.

Tayman, J. 1996. The accuracy of small area population forecasts based on a spatial interaction land use modeling system. Journal of the American Planning Association 62: 85-98.

Tayman, J., E. Schafer, and L. Carter. 1998. The role of population size in the determination and prediction of population forecast errors: An evaluation using confidence intervals for subcounty areas. Population Research and Policy Review 17: 1-20.

Tayman, J., Swanson, D.A. \& Barr, C.F. (1999). In search of the ideal measure of accuracy for subnational demographic forecasts, Population Research and Policy Review 18: 387-409.

Webby, R., and M. O’Connor. 1996. Judgemental and statistical time series forecasting: A review of the literature. International Journal of Forecasting 12: 91-118.

White, H. R. (1954). Empirical study of the accuracy of selected methods for projecting state populations. Journal of the American Statistical Association, 49, 480-498.

Table 1a. MAPE by Horizon and Population Size

| Horizon | Population Size | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | $<1,000$ | 31.7 | 34.8 | 43.8 | 45.8 | 35.3 | 21.3 | 30.8 | 30.6 |
| 10 | 1,000 to 2,000 | 19.5 | 21.3 | 27.4 | 30.8 | 24.6 | 18.2 | 20.0 | 19.5 |
| 10 | 2,000 to 5,000 | 14.3 | 15.8 | 19.1 | 21.1 | 22.1 | 15.9 | 14.4 | 14.4 |
| 10 | 5,000 to 10,000 | 13.4 | 14.2 | 18.0 | 19.8 | 21.9 | 17.7 | 12.8 | 13.0 |
| 10 | 10,000 to 25,000 | 11.2 | 12.9 | 17.2 | 32.9 | 19.1 | 19.3 | 13.1 | 11.7 |
| 10 | $>25,000$ | 10.0 | 11.9 | 15.4 | 26.1 | 14.0 | 20.9 | 11.2 | 10.7 |
| 20 | $<1,000$ | 55.9 | 68.7 | 92.9 | 325.8 | 79.1 | 31.8 | 96.6 | 59.1 |
| 20 | 1,000 to 2,000 | 33.8 | 44.1 | 63.5 | 193.1 | 52.1 | 29.1 | 60.1 | 39.1 |
| 20 | 2,000 to 5,000 | 26.9 | 35.4 | 43.0 | 110.6 | 49.8 | 25.9 | 38.5 | 29.5 |
| 20 | 5,000 to 10,000 | 22.9 | 26.8 | 36.5 | 53.3 | 47.8 | 31.0 | 23.6 | 22.4 |
| 20 | 10,000 to 25,000 | 19.7 | 29.8 | 46.2 | 472.8 | 42.9 | 31.3 | 93.6 | 24.3 |
| 20 | $>25,000$ | 17.4 | 28.5 | 44.8 | 221.2 | 32.7 | 35.0 | 50.6 | 24.1 |

Table 1b. MALPE by Horizon and Population Size

| Horizon | Population Size | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | $<1,000$ | 0.7 | 3.0 | -7.4 | 20.6 | 26.3 | -3.9 | 6.5 | 4.4 |
| 10 | 1,000 to 2,000 | -0.1 | 2.7 | -3.7 | 14.7 | 13.7 | -12.3 | 2.5 | 1.5 |
| 10 | 2,000 to 5,000 | -0.3 | 2.5 | -4.0 | 10.7 | 12.9 | -11.7 | 1.7 | 0.9 |
| 10 | 5,000 to 10,000 | -2.4 | 0.4 | -5.9 | 9.5 | 9.5 | -15.3 | -0.7 | -1.3 |
| 10 | 10,000 to 25,000 | -0.4 | 3.9 | 1.0 | 26.6 | 7.7 | -18.5 | 3.4 | 1.5 |
| 10 | $>25,000$ | 0.3 | 5.4 | 5.0 | 21.3 | 6.3 | -20.3 | 3.0 | 3.2 |
| 20 | $<1,000$ | 12.2 | 23.7 | 9.1 | 291.8 | 66.4 | -9.1 | 65.7 | 22.7 |
| 20 | 1,000 to 2,000 | 7.8 | 20.7 | 9.4 | 176.7 | 37.5 | -22.4 | 38.3 | 15.9 |
| 20 | 2,000 to 5,000 | 3.5 | 15.3 | -4.5 | 96.2 | 37.1 | -19.9 | 21.3 | 9.3 |
| 20 | 5,000 to 10,000 | -5.4 | 4.9 | -15.5 | 38.1 | 25.9 | -27.8 | 3.4 | -0.5 |
| 20 | 10,000 to 25,000 | 1.0 | 17.4 | 7.0 | 464.8 | 28.6 | -30.7 | 81.4 | 10.9 |
| 20 | $>25,000$ | 1.7 | 20.6 | 18.5 | 215.6 | 23.4 | -34.5 | 40.9 | 14.4 |

Table 2a. MAPE by Horizon and Growth Rate

| Horizon | Growth Rate | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | $<-10 \%$ | 36.3 | 39.4 | 54.8 | 28.8 | 27.7 | 17.0 | 26.2 | 28.6 |
| 10 | $-10 \%$ to $0 \%$ | 12.7 | 13.0 | 26.2 | 12.5 | 23.9 | 10.7 | 11.1 | 12.0 |
| 10 | $0 \%$ to $10 \%$ | 10.6 | 10.8 | 16.6 | 10.7 | 24.4 | 10.8 | 10.7 | 10.6 |
| 10 | $10 \%$ to $25 \%$ | 12.0 | 12.8 | 14.0 | 12.7 | 22.0 | 14.2 | 12.1 | 11.9 |
| 10 | $25 \%$ to $50 \%$ | 15.3 | 17.3 | 17.4 | 19.6 | 20.2 | 20.2 | 15.2 | 15.9 |
| 10 | $>50 \%$ | 22.1 | 26.2 | 31.3 | 68.0 | 23.2 | 32.9 | 26.1 | 23.9 |
| 20 | $<-10 \%$ | 62.3 | 70.1 | 88.4 | 48.9 | 83.9 | 26.7 | 41.4 | 49.6 |
| 20 | $-10 \%$ to $0 \%$ | 17.5 | 18.9 | 57.8 | 16.9 | 53.5 | 14.7 | 15.1 | 16.5 |
| 20 | $0 \%$ to $10 \%$ | 15.1 | 16.0 | 40.4 | 15.4 | 52.2 | 15.5 | 15.5 | 14.9 |
| 20 | $10 \%$ to $25 \%$ | 20.1 | 24.3 | 32.0 | 22.7 | 51.4 | 22.9 | 21.1 | 20.3 |
| 20 | $25 \%$ to $50 \%$ | 27.2 | 37.6 | 34.3 | 43.5 | 49.0 | 30.9 | 29.2 | 30.5 |
| 20 | $>50 \%$ | 41.4 | 59.2 | 80.8 | 604.6 | 45.5 | 50.8 | 130.7 | 52.5 |

Table 2b. MALPE by Horizon and Growth Rate

| Horizon | Growth Rate | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | $<-10 \%$ | -34.8 | -38.0 | -53.9 | -27.0 | 17.5 | -5.9 | -23.7 | -26.4 |
| 10 | $-10 \%$ to $0 \%$ | -8.5 | -9.1 | -23.9 | -8.2 | 19.9 | -4.0 | -5.6 | -7.5 |
| 10 | $0 \%$ to $10 \%$ | -0.6 | 0.0 | -11.7 | -0.4 | 20.5 | -4.9 | 0.5 | -1.2 |
| 10 | $10 \%$ to $25 \%$ | 2.7 | 4.9 | -4.0 | 5.0 | 15.9 | -10.1 | 2.4 | 2.6 |
| 10 | $25 \%$ to $50 \%$ | 5.8 | 10.2 | 6.2 | 13.8 | 11.7 | -16.2 | 5.3 | 7.5 |
| 10 | $>50 \%$ | 6.8 | 16.0 | 23.1 | 64.3 | 1.8 | -28.1 | 14.0 | 11.8 |
| 20 | $<-10 \%$ | -57.5 | -66.2 | -88.4 | -43.0 | 71.2 | -6.6 | -31.7 | -43.3 |
| 20 | $-10 \%$ to $0 \%$ | -14.1 | -16.1 | -56.5 | -13.4 | 49.3 | -5.8 | -9.4 | -12.3 |
| 20 | $0 \%$ to $10 \%$ | -2.2 | 0.3 | -37.0 | -1.4 | 47.8 | -10.4 | -0.5 | -3.0 |
| 20 | $10 \%$ to $25 \%$ | 3.3 | 10.9 | -19.0 | 9.7 | 40.0 | -19.4 | 4.3 | 3.9 |
| 20 | $25 \%$ to $50 \%$ | 13.2 | 28.6 | 9.4 | 36.8 | 38.4 | -25.4 | 16.8 | 19.4 |
| 20 | $>50 \%$ | 16.4 | 46.2 | 69.5 | 601.3 | 17.9 | -42.4 | 118.2 | 37.2 |

Table 3a. MAPE by Horizon and Population Size and Growth Rate

| Horizon | Size | Growth Rate | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | $<2,000$ | $<0 \%$ | 29.8 | 32.0 | 48.2 | 24.8 | 30.3 | 17.2 | 23.1 | 24.8 |
| 10 | 2,000 to 10,000 | $<0 \%$ | 13.2 | 13.8 | 24.0 | 12.2 | 18.4 | 8.9 | 10.2 | 11.5 |
| 10 | $>10,000$ | $<0 \%$ | 10.5 | 11.1 | 24.7 | 10.0 | 22.2 | 6.2 | 7.9 | 9.2 |
| 10 | $<2,000$ | $0 \%$ to $50 \%$ | 19.1 | 20.3 | 23.1 | 21.1 | 29.4 | 17.3 | 19.0 | 19.4 |
| 10 | 2,000 to 10,000 | $0 \%$ to $50 \%$ | 11.6 | 12.6 | 14.9 | 13.1 | 21.9 | 13.6 | 11.8 | 11.8 |
| 10 | $>10,000$ | $0 \%$ to $50 \%$ | 8.8 | 9.7 | 11.4 | 10.6 | 16.8 | 14.8 | 8.8 | 8.9 |
| 10 | $<2,000$ | $>50 \%$ | 43.8 | 53.0 | 59.7 | 107.1 | 43.4 | 33.8 | 52.1 | 48.6 |
| 10 | 2,000 to 10,000 | $>50 \%$ | 21.9 | 24.2 | 26.6 | 55.6 | 24.8 | 34.8 | 23.0 | 22.5 |
| 10 | $>10,000$ | $>50 \%$ | 13.4 | 16.6 | 21.9 | 57.4 | 14.9 | 31.6 | 17.1 | 14.9 |
| 20 | $<2,000$ | $<0 \%$ | 48.6 | 54.3 | 81.2 | 40.6 | 84.7 | 25.3 | 34.9 | 41.0 |
| 20 | 2,000 to 10,000 | $<0 \%$ | 23.6 | 25.7 | 55.6 | 19.3 | 43.4 | 16.7 | 17.4 | 19.3 |
| 20 | $>10,000$ | $<0 \%$ | 17.5 | 20.1 | 59.5 | 15.3 | 49.3 | 5.6 | 12.5 | 14.3 |
| 20 | $<2,000$ | $0 \%$ to $50 \%$ | 29.1 | 35.8 | 45.2 | 37.5 | 61.9 | 24.4 | 30.4 | 30.5 |
| 20 | 2,000 to 10,000 | $0 \%$ to $50 \%$ | 20.7 | 26.3 | 31.9 | 27.3 | 49.2 | 22.1 | 22.1 | 22.2 |
| 20 | $>10,000$ | $0 \%$ to $50 \%$ | 14.9 | 19.8 | 28.8 | 21.1 | 42.0 | 24.7 | 15.9 | 15.8 |
| 20 | $<2,000$ | $>50 \%$ | 78.2 | 103.1 | 140.1 | 818.0 | 67.4 | 52.5 | 200.6 | 95.4 |
| 20 | 2,000 to 10,000 | $>50 \%$ | 38.2 | 47.7 | 53.0 | 273.6 | 50.6 | 54.6 | 64.9 | 41.0 |
| 20 | $>10,000$ | $>50 \%$ | 22.3 | 39.9 | 60.5 | 649.1 | 30.2 | 47.8 | 124.1 | 33.9 |

Table 3b. MALPE by Horizon and Population Size and Growth Rate

| Horizon | Size | Growth Rate | LIN | SHR | SFT | EXP | COS | CON | AV | TAV |
| :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | $<2,000$ | $<0 \%$ | -25.7 | -28.1 | -45.7 | -20.5 | 20.6 | -5.9 | -17.6 | -20.0 |
| 10 | 2,000 to 10,000 | $<0 \%$ | -11.5 | -12.2 | -23.3 | -10.3 | 15.9 | -3.2 | -7.4 | -9.3 |
| 10 | $>10,000$ | $<0 \%$ | -9.8 | -10.5 | -24.7 | -9.2 | 19.7 | -4.3 | -6.5 | -8.4 |
| 10 | $<2,000$ | $0 \%$ to $50 \%$ | 6.2 | 8.7 | -2.2 | 9.4 | 22.9 | -6.4 | 6.4 | 6.4 |
| 10 | 2,000 to 10,000 | $0 \%$ to $50 \%$ | 2.0 | 4.4 | -3.7 | 5.3 | 15.5 | -10.3 | 2.2 | 2.3 |
| 10 | $>10,000$ | $0 \%$ to $50 \%$ | 0.4 | 3.0 | -3.1 | 4.6 | 11.0 | -14.0 | 0.3 | 0.9 |
| 10 | $<2,000$ | $>50 \%$ | 28.8 | 40.8 | 48.8 | 100.5 | 23.3 | -13.7 | 38.1 | 35.3 |
| 10 | 2,000 to 10,000 | $>50 \%$ | -1.6 | 6.0 | 10.6 | 49.3 | -8.0 | -33.8 | 3.7 | 1.5 |
| 10 | $>10,000$ | $>50 \%$ | 1.6 | 10.7 | 18.2 | 55.9 | -1.9 | -31.4 | 8.8 | 7.1 |
| 20 | $<2,000$ | $<0 \%$ | -41.8 | -48.6 | -79.9 | -33.0 | 72.3 | -8.2 | -23.2 | -32.9 |
| 20 | 2,000 to 10,000 | $<0 \%$ | -22.2 | -24.9 | -55.6 | -17.6 | 40.3 | -2.6 | -13.7 | -16.8 |
| 20 | $>10,000$ | $<0 \%$ | -17.4 | -19.9 | -59.5 | -15.1 | 49.3 | -4.2 | -11.1 | -14.2 |
| 20 | $<2,000$ | $0 \%$ to $50 \%$ | 10.9 | 20.8 | -11.7 | 22.5 | 53.3 | -14.4 | 13.6 | 13.4 |
| 20 | 2,000 to 10,000 | $0 \%$ to $50 \%$ | 4.8 | 13.4 | -12.3 | 15.1 | 39.2 | -18.6 | 6.9 | 6.7 |
| 20 | $>10,000$ | $0 \%$ to $50 \%$ | 0.8 | 9.8 | -17.3 | 12.4 | 33.5 | -23.7 | 2.6 | 3.1 |
| 20 | $<2,000$ | $>50 \%$ | 53.6 | 85.7 | 126.4 | 813.0 | 39.3 | -24.3 | 182.3 | 76.2 |
| 20 | 2,000 to 10,000 | $>50 \%$ | -3.1 | 22.3 | 27.3 | 265.7 | 2.4 | -52.2 | 43.7 | 11.4 |
| 20 | $>10,000$ | $>50 \%$ | 4.7 | 34.9 | 57.4 | 648.9 | 12.8 | -47.7 | 118.5 | 27.2 |

Table 4a. MAPE by Horizon and Technique (Subcounty Data Set)

| Horizon | LIN | SHR | SFT | EXP | COS | CON | AV | TAV | C1 | C2 | C3 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 5 | 9.9 | 10.4 | 12.1 | 12.7 | 11.9 | 11.5 | 9.5 | 9.6 | 8.8 | 9.0 | 8.7 |
| 10 | 17.1 | 19.0 | 24.1 | 30.0 | 23.0 | 19.1 | 17.4 | 17.1 | 14.7 | 15.5 | 14.4 |
| 15 | 23.6 | 28.5 | 38.9 | 71.2 | 35.6 | 25.2 | 28.9 | 24.7 | 19.4 | 21.1 | 19.0 |
| 20 | 30.5 | 40.2 | 56.1 | 231.5 | 51.8 | 31.0 | 61.6 | 34.1 | 24.7 | 27.9 | 24.1 |
| 25 | 41.4 | 64.3 | 89.3 | $1,216.7$ | 80.9 | 36.1 | 238.0 | 53.5 | 30.9 | 38.8 | 30.3 |

Table 4b. MALPE by Horizon and Technique (Subcounty Data Set)

| Horizon | LIN | SHR | SFT | EXP | COS | CON | AV | TAV | C1 | C2 | C3 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 5 | -0.6 | 0.6 | -1.4 | 4.9 | 5.2 | -7.6 | 0.2 | 0.2 | -1.6 | 0.9 | -1.3 |
| 10 | -0.2 | 3.3 | -2.2 | 17.8 | 13.2 | -13.5 | 3.1 | 2.1 | -2.6 | 2.4 | -2.0 |
| 15 | 0.4 | 7.7 | -2.4 | 55.4 | 23.4 | -19.1 | 10.9 | 5.2 | -3.9 | 4.1 | -3.0 |
| 20 | 3.9 | 17.6 | 4.7 | 215.3 | 37.8 | -23.5 | 42.6 | 12.8 | -4.5 | 7.8 | -3.3 |
| 25 | 12.4 | 42.1 | 28.9 | $1,201.4$ | 67.0 | -28.5 | 220.6 | 31.7 | -3.4 | 16.1 | -2.3 |

Note: Composite forecasts were created from the following techniques

$$
\begin{aligned}
\text { C1 } & =\text { CON when size }<2,000 \\
& =\text { LIN when size }>2,000
\end{aligned}
$$

$$
\begin{aligned}
\text { C2 } & =\text { CON when growth rate }<0 \% \\
& =\text { LIN when growth rate }>0 \%
\end{aligned}
$$

$$
\begin{aligned}
\text { C3 } & =\text { CON when growth rate }<0 \% \\
& =\text { CON when growth rate }>0 \% \text { and size }<2,000 \\
& =\text { LIN when growth rate }>0 \% \text { and size }>2,000
\end{aligned}
$$

Table 5a. MAPE by Horizon and Technique (County Data Set)

| Horizon | LIN | SHR | SFT | EXP | COS | CON | AV | TAV | C1 | C2 | C3 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | 10.5 | 10.9 | 13.0 | 11.7 | 14.3 | 11.6 | 10.1 | 10.1 | 10.3 | 10.3 | 10.2 |
| 20 | 19.7 | 21.1 | 28.2 | 27.2 | 27.8 | 19.7 | 19.2 | 18.7 | 18.6 | 18.5 | 18.0 |
| 30 | 31.0 | 34.0 | 46.9 | 79.7 | 43.8 | 27.6 | 35.7 | 29.2 | 28.3 | 28.2 | 26.9 |

Table 5b. MALPE by Horizon and Technique (County Data Set)

| Horizon | LIN | SHR | SFT | EXP | COS | CON | AV | TAV | C1 | C2 | C3 |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10 | -1.6 | -1.4 | -4.2 | 2.1 | 7.3 | -4.1 | -0.3 | -0.7 | -0.7 | 1.8 | 0.9 |
| 20 | -3.2 | -2.7 | -10.9 | 10.0 | 17.6 | -6.5 | 0.7 | -0.8 | -1.1 | 4.3 | 2.5 |
| 30 | -5.2 | -4.3 | -19.1 | 53.2 | 30.0 | -7.9 | 7.8 | -0.8 | -2.0 | 6.8 | 4.1 |

Note: Composite forecasts were created from the following techniques

$$
\begin{aligned}
\text { C1 } & =\text { CON when size }<10,000 \\
& =\text { LIN when size }>10,000 \\
\text { C2 } & =\text { CON when growth rate }<0 \% \\
& =\text { LIN when growth rate }>0 \% \\
\text { C3 } & =\text { CON when growth rate }<0 \% \\
& =\text { CON when growth rate }>0 \% \text { and size }<10,000 \\
& =\text { LIN when growth rate }>0 \% \text { and size }>10,000
\end{aligned}
$$

Table 6. Percentage Point Difference in MAPEs, Accounting for Special Populations

|  |  | SP1 |  |  |  |  | SP2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Horizon | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ |  |  |
| 1985 | 5 | 0.0 | 0.0 | 0.0 | 0.0 | -0.1 | -0.2 | -0.3 | -0.3 |  |  |
| 1990 | 5 | 0.0 | 0.0 | 0.0 | 0.0 | -0.3 | -0.8 | -1.0 | -1.2 |  |  |
| 1995 | 5 | 0.1 | 0.3 | 0.5 | 0.7 | -0.9 | -2.0 | -2.7 | -3.3 |  |  |
| 2000 | 5 | -0.3 | -0.8 | -1.0 | -1.3 | -0.6 | -1.3 | -1.7 | -2.2 |  |  |
| 2005 | 5 | 0.0 | -0.1 | -0.1 | -0.2 | -0.5 | -1.2 | -1.7 | -2.1 |  |  |
| 1990 | 10 | -0.2 | -0.4 | -0.6 | -0.7 | -0.6 | -1.5 | -1.9 | -2.2 |  |  |
| 1995 | 10 | 0.0 | 0.1 | 0.1 | 0.3 | -1.1 | -2.7 | -3.5 | -4.2 |  |  |
| 2000 | 10 | 0.2 | 0.5 | 0.7 | 1.1 | -1.7 | -4.0 | -5.3 | -6.3 |  |  |
| 2005 | 10 | 0.3 | 0.5 | 0.8 | 0.6 | -0.3 | -0.7 | -0.9 | -1.4 |  |  |
| 1995 | 15 | 0.0 | 0.0 | 0.0 | 0.0 | -0.3 | -0.8 | -1.0 | -1.0 |  |  |
| 2000 | 15 | 0.0 | 0.0 | 0.1 | 0.3 | -1.2 | -2.8 | -3.7 | -4.3 |  |  |
| 2005 | 15 | 0.2 | 0.4 | 0.6 | 1.0 | -1.7 | -3.8 | -5.0 | -6.0 |  |  |
| 2000 | 20 | 0.0 | 0.1 | 0.1 | 0.1 | -0.5 | -1.3 | -1.6 | -2.0 |  |  |
| 2005 | 20 | -0.3 | -0.8 | -1.1 | -1.0 | -1.6 | -3.8 | -5.1 | -5.8 |  |  |
| 2005 | 25 | -0.1 | -0.3 | -0.4 | -0.5 | -0.4 | -0.9 | -1.2 | -1.6 |  |  |
| All | 5 | 0.0 | -0.1 | -0.1 | -0.2 | -0.5 | -1.1 | -1.5 | -1.8 |  |  |
| All | 10 | 0.1 | 0.2 | 0.3 | 0.3 | -0.9 | -2.2 | -2.9 | -3.5 |  |  |
| All | 15 | 0.1 | 0.2 | 0.2 | 0.4 | -1.1 | -2.5 | -3.3 | -3.8 |  |  |
| All | 20 | -0.1 | -0.4 | -0.5 | -0.5 | -1.1 | -2.5 | -3.4 | -3.9 |  |  |
| All | 25 | -0.1 | -0.3 | -0.4 | -0.5 | -0.4 | -0.9 | -1.2 | -1.6 |  |  |

Note
This table is restricted to the subset of subcounty areas with special populations ( $\mathrm{N}=141$ ).
Columns titled " $1 \%, 2.5 \%, 5 \%$ " further restrict the analysis to subcounty areas where the special population exceeds $1 \%(\mathrm{~N}=61)$, $2.5 \%(\mathrm{~N}=45)$, and $5 \%(\mathrm{~N}=36)$ of total population.

SP1 = Accounts for special populations by holding them constant at the launch year value.
SP2 = Accounts for special populations using the actual target year value.

Table 7. Percentage Point Difference in MAPEs, Accounting for Annexations (Incorporated Places)

|  |  | A1 |  |  |  |  |  |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Year | Horizon | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ |
| 1985 | 5 | 0.0 | 0.1 | 0.1 | 0.1 | 0.2 | 0.3 | 0.3 | 0.5 |
| 1990 | 5 | -0.3 | -0.4 | -0.5 | -0.7 | -0.8 | -1.1 | -1.4 | -2.0 |
| 1995 | 5 | -0.1 | -0.2 | -0.2 | -0.2 | -0.2 | -0.3 | -0.4 | -0.5 |
| 2000 | 5 | -0.1 | -0.2 | -0.3 | -0.4 | -0.5 | -0.7 | -0.8 | -1.1 |
| 2005 | 5 | 0.2 | 0.3 | 0.3 | 0.3 | -1.3 | -1.8 | -2.3 | -3.2 |
| 1990 | 10 | -0.4 | -0.6 | -0.7 | -0.9 | -0.7 | -1.0 | -1.1 | -1.5 |
| 1995 | 10 | -0.7 | -1.0 | -1.3 | -1.8 | -1.7 | -2.3 | -3.0 | -4.2 |
| 2000 | 10 | 0.0 | -0.1 | -0.1 | -0.1 | -0.2 | -0.2 | -0.3 | -0.3 |
| 2005 | 10 | 0.2 | 0.3 | 0.3 | 0.3 | -2.0 | -2.8 | -3.5 | -4.6 |
| 1995 | 15 | -1.0 | -1.4 | -1.7 | -2.4 | -1.3 | -1.9 | -2.3 | -3.2 |
| 2000 | 15 | -1.1 | -1.6 | -2.0 | -2.7 | -2.2 | -3.0 | -3.8 | -5.1 |
| 2005 | 15 | 0.2 | 0.3 | 0.4 | 0.6 | -1.3 | -1.8 | -2.2 | -3.0 |
| 2000 | 20 | -1.4 | -1.9 | -2.4 | -3.2 | -1.6 | -2.2 | -2.8 | -3.8 |
| 2005 | 20 | -1.2 | -1.6 | -2.1 | -2.9 | -3.3 | -4.6 | -5.9 | -8.0 |
| 2005 | 25 | -1.5 | -2.1 | -2.6 | -3.5 | -2.4 | -3.4 | -4.3 | -5.8 |
| All | 5 | -0.1 | -0.1 | -0.1 | -0.2 | -0.5 | -0.7 | -0.9 | -1.3 |
| All | 10 | -0.3 | -0.4 | -0.4 | -0.6 | -1.1 | -1.6 | -2.0 | -2.6 |
| All | 15 | -0.6 | -0.9 | -1.1 | -1.5 | -1.6 | -2.2 | -2.8 | -3.8 |
| All | 20 | -1.3 | -1.8 | -2.3 | -3.0 | -2.5 | -3.4 | -4.3 | -5.9 |
| All | 25 | -1.5 | -2.1 | -2.6 | -3.5 | -2.4 | -3.4 | -4.3 | -5.8 |

Note
This table is restricted to the subset of incorporated places with annexations ( $\mathrm{N}=183$ ).
Columns titled " $1 \%, 2.5 \%, 5 \%$ " further restrict the analysis to incorporated places where the annexed population exceeds $1 \%(\mathrm{~N}=131)$, $2.5 \%(\mathrm{~N}=100)$, and $5 \%(\mathrm{~N}=71)$ of total population.

A1 = Accounts for annexations that occurred between the base year and launch year.
A2 = Accounts for annexations that occurred between the base year and target year.

Table 8. Percentage Point Difference in MAPEs, Accounting for Annexations (Unincorporated Areas)

|  |  | A1 |  |  |  |  |  |  |  |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Year | Horizon | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ | All | $>1 \%$ | $>2.5 \%$ | $>5 \%$ |  |
| 1985 | 5 | 0.0 | 0.1 | 0.1 | 0.8 | -0.6 | -1.1 | -1.7 | -6.6 |  |
| 1990 | 5 | 0.0 | -0.1 | -0.1 | -0.9 | 0.1 | 0.2 | 0.1 | -0.4 |  |
| 1995 | 5 | -0.1 | -0.1 | -0.3 | -2.7 | -0.2 | -0.4 | -0.7 | -3.9 |  |
| 2000 | 5 | 0.0 | 0.1 | 0.1 | -0.1 | -0.1 | -0.1 | -0.1 | 0.1 |  |
| 2005 | 5 | 0.1 | 0.2 | 0.3 | 2.1 | -0.6 | -1.0 | -1.9 | -7.1 |  |
| 1990 | 10 | 0.3 | 0.7 | 1.0 | 5.3 | -0.1 | -0.2 | -0.5 | -1.0 |  |
| 1995 | 10 | -0.4 | -0.8 | -1.2 | -4.4 | -0.4 | -0.8 | -1.3 | -5.5 |  |
| 2000 | 10 | -0.3 | -0.5 | -0.8 | -4.9 | -0.3 | -0.5 | -0.9 | -2.8 |  |
| 2005 | 10 | 0.2 | 0.3 | 0.5 | 2.4 | -0.2 | -0.4 | -0.6 | -2.5 |  |
| 1995 | 15 | 0.6 | 1.2 | 1.7 | 7.4 | -0.2 | -0.4 | -0.9 | -0.7 |  |
| 2000 | 15 | -0.6 | -1.2 | -1.8 | -6.3 | -0.5 | -1.0 | -1.6 | -6.5 |  |
| 2005 | 15 | -0.2 | -0.4 | -0.8 | -4.4 | -0.5 | -0.9 | -1.6 | -5.5 |  |
| 2000 | 20 | 0.7 | 1.3 | 2.3 | 8.8 | -0.2 | -0.4 | -0.5 | 0.5 |  |
| 2005 | 20 | -0.1 | -0.3 | -0.3 | 0.9 | -0.7 | -1.4 | -2.6 | -11.4 |  |
| 2005 | 25 | 1.2 | 2.3 | 3.7 | 13.9 | -0.3 | -0.6 | -1.5 | -4.4 |  |
| All | 5 | 0.0 | 0.0 | 0.0 | -0.2 | -0.3 | -0.5 | -0.9 | -3.6 |  |
| All | 10 | 0.0 | -0.1 | -0.1 | -0.4 | -0.3 | -0.5 | -0.8 | -2.9 |  |
| All | 15 | -0.1 | -0.1 | -0.3 | -1.1 | -0.4 | -0.8 | -1.4 | -4.3 |  |
| All | 20 | 0.3 | 0.5 | 1.0 | 4.9 | -0.5 | -0.9 | -1.6 | -5.5 |  |
| All | 25 | 1.2 | 2.3 | 3.7 | 13.9 | -0.3 | -0.6 | -1.5 | -4.4 |  |

Note
This table is restricted to the subset of unincorporated areas with annexations ( $\mathrm{N}=51$ ).
Columns titled "1\%, $2.5 \%, 5 \%$ " further restrict the analysis to unincorporated areas where the annexed population exceeds $1 \%(\mathrm{~N}=27), 2.5 \%(\mathrm{~N}=17)$, and $5 \%(\mathrm{~N}=4)$ of total population.

A1 = Accounts for annexations that occurred between the base year and launch year.
A2 = Accounts for annexations that occurred between the base year and target year.

