Decomposing lifespan inequality by subgroup, with an illustration on sex differences in mortality

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Introduction

The human lifespan has been increasing at an astounding 2.5 years per decade since 1840 for females in the best-practice country [1]. Concomitant to gains in longevity has been the remarkable decline in life disparity. While in 1840 people were dying with an average remaining life expectancy of 24 years, in today's most advanced countries this life disparity has fallen to around 9 years [2].

Despite progress at reducing mortality, systematic gaps in life expectancy persist for socioeconomic groupings, areas, race, and gender. Moreover, groups disadvantaged in life expectancy tend also to have higher dispersion in the age at death [3, 4]. While it is clear that inequality exists between subgroups, a question arises: How much do the between-subgroup lifespan inequalities account for the total lifespan inequality?

In this study I show how the contribution of between-subgroup inequality to total inequality in lifespan can be measured using a decomposition of Theil's entropy index. I illustrate this method by estimating the contribution of between-sex lifespan inequality to total lifespan inequality for all countries and years of the Human Mortality Database[2] (HMD).

Methods

Preparing death distributions

Data from the HMD is designed so as to be optimal for each population (male, female, both sexes combined) but without the explicit aim of consistency between the three sets of life tables. Thus calculating and decomposing inequality from the given total, male and female life tables as given will involve a slight residual. To get around this (resulting in exact decompositions), I created my own life tables (ages 0-110+), using the lifetable male and female death rates (m_x) from

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[2] Data available freely at www.mortality.org

the HMD, and the average number of life years lived in the interval by those who die (a_x) from the both sexes combined life table. Since more males are born into a population than females, I adjusted the population radix by the sex ratio at birth. The resultant life table male and female death density (d_x) values are the male and female death distributions I use for the calculation and decomposition of lifespan inequality. The d_x of the total population is obtained by summing the male and female d_x. Finally the average age at death (e_0) values used for all populations were obtained using an alternate formulation of life expectancy, namely the product of the average age at death and the number of deaths, divided by the total deaths.

Measuring and decomposing inequality

I measure inequality in age at death using Theil's entropy index. Theil derived his index from information theory to measure the degree of disorder in a distribution, a closely related concept to inequality [5]. While it has no direct demographic interpretation, it correlates closely to other measures such as the Gini coefficient and the standard deviation [4], however it is known to be more sensitive to changes in early parts of the distribution than other measures [6]. It also possesses many of the desirable traits of an inequality measure including scale independence, population-size independence, satisfying the Pigou-Dalton principle of transfers, and most important to this study—decomposability.

Given requirements of scale independence and population-size independence Shorrocks [7, 8] showed that only the single parameter Generalised Entropy family can be additively decomposable into its between-group (*BG*) and within-group (*WG*) components, of which Theil's index is best known. The mean logarithmic deviation also fits into this family, but is even more sensitive to changes in the early part of the distribution, which might make it less suitable for mortality research because of the large historical fluctuations in infant mortality. The Gini coefficient can also be decomposed into between- and within-group inequality, but results in an overlap term anytime the subgroup distributions overlap. Although some researchers have used the Gini decomposition [9, 10] or the related Health Concentration Index [11] to determine the degree of inequality in age-standardised levels of health explained by socioeconomic status, the much larger degree of overlap in subgroup death distributions as compared to these health-related quality of life indices make the Gini decomposition an unattractive choice in mortality research. Moreover, as Cowell [12] shows it is possible for the Gini in extreme circumstances to register an increase in inequality in every population subgroup while at the same time showing an overall decrease in the level on inequality, thus proving the impossibility of formulating inequality change as being a function of inequality change in the component subgroups.

Theil's entropy index *(T)* can be reasonably estimated from the life table death density by the following equation [4],

$$T = \frac{1}{l_0}\sum_{x=0}^{\omega} d_x\left[\left(\frac{\psi_x}{e_0}\right)\ln\left(\frac{\psi_x}{e_0}\right)\right] \tag{1}$$

where 0 and ω are respectively the youngest and oldest age intervals in the life table, l_0 is the radix of the population (taken to be the initial subgroup population size), d_x and ψ_x are respectively the life table number of deaths and the average age at death in the age interval x to $x+1$, and e_0 is the average age at death for the life table population. The greater the value of the index, the greater is the level of disorder or inequality. A value of 0 would indicate perfect equality (i.e. everyone died at the same age).

Theil's index is then decomposed into its between- and within-group components, in the simple fashion $T = BG + WG$. Calculating between-group inequality can be done by assuming that everyone in subgroup i has that group's mean age at death weighted by the subgroup's population share (w^i).

$$BG = \sum_{i=1}^{n}\left[w^i\left(\frac{e_0^i}{e_0^t}\right)\ln\left(\frac{e_0^i}{e_0^t}\right)\right] \tag{2}$$

The between-group inequality component captures the level of inequality in the age at death which would be experienced by the whole population if each person in a subgroup died at the subgroup life expectancy, while within-group inequality is a weighted average of the subgroup inequality levels. In this case n is the number of subgroups, e_0^i refers to the mean age at death for subgroup i, and e_0^t is the mean age at death of the total population. Within-group inequality is a weighted average of the inequality levels present within each subgroup calculated by,

$$WG = \sum_{i=1}^{n}\left[w^i T^i\left(\frac{e_0^i}{e_0^t}\right)\right] \tag{3}$$

where T^i is the subgroup i Theil index of inequality.

Results

The results of the decomposition for a selected group of countries are presented for 2005 in Table 1, and for all countries and time periods in Figure 1. Not surprisingly, countries with a higher sex gap also had a higher between-group component, and higher between-group contributions to total inequality (Fig 1A and 1C). The within-group component showed little association to the life expectancy gap, except for in the most unequal countries (Fig 1B).

	life expectancy gap (years)	Theil's index * 100			Decomposition		
	Female-Male	Female	Male	Total	WG	BG	BG/Theil (%)
England	4.18	1.83	2.36	2.13	2.096	0.035	1.631
Sweden	4.33	1.48	1.88	1.72	1.680	0.036	2.103
Denmark	4.51	1.78	2.36	2.11	2.068	0.042	1.973
Canada	4.68	1.94	2.54	2.28	2.237	0.043	1.871
USA	5.15	2.45	3.38	2.97	2.911	0.055	1.851
Chile	5.75	2.35	3.41	2.94	2.873	0.068	2.323
Taiwan	6.23	2.17	3.28	2.81	2.729	0.081	2.879
Czech Republic	6.37	1.67	2.52	2.18	2.091	0.088	4.017
France	6.77	1.76	2.60	2.26	2.172	0.092	4.063
Japan	6.95	1.55	2.13	1.93	1.836	0.090	4.667
Hungary	8.47	2.30	3.35	2.98	2.811	0.169	5.674
Poland	8.55	2.14	3.34	2.88	2.719	0.163	5.649
Belarus	12.21	2.69	4.61	3.98	3.590	0.394	9.882
Russia	13.54	3.74	6.13	5.38	4.846	0.535	9.935

Table 1: Theil's index of inequality and its decomposition for a select group of countries, HMD, 2005, ordered by the life expectancy gap between the sexes

At lower levels of life expectancy the between-group component is higher given the same sex gap (Fig.1B). This is because the between-group component captures proportional life expectancy differences rather than the absolute differences captured in the sex gap. Moreover, changes to the subgroup distributions themselves can affect the life expectancy gap. As Vaupel and Canudas Romo [13] derived, the change in life expectancy is equal to the product of life disparity and the rate of progress at reducing age-specific death rates. Therefore subgroups with higher life disparity will experience larger gains in life expectancy than more egalitarian subgroups, given the same progress in mortality reduction. Analysis of the recent narrowing of the sex gap has shown it to been driven in large part by the effect of a greater dispersion in male age at death as opposed to declining sex ratios in mortality [14].

Moreover, for any given sex gap, the range of the inequality explained by the between-group component varies. For example, the between-group component explains between 0.5 and 3.3 percent of the total inequality for life expectancy gaps of between 5-6 years; for gaps of 10-11 years it is between 2.5 and 8.3 percent. Mostly this is because of differences in the within-group inequality component. Since total inequality is the sum of between-group and within-group inequality, lowering the within-group inequality in even just one subgroup acts to lower the total inequality, causing between-group inequality to explain more of the total lifespan inequality. Historically, the within-group inequality levels were much higher than they are today; consequently the larger denominator meant that the between-group component explained less (Fig. 1D). This also makes clear why as countries have begun to lower their sex differential in life expectancy, the contribution of between-group inequality has not declined back to historical levels reached at similar sex gaps. It also points to the importance of examining the within-group component in explaining how sex is differentially affecting the survivorship of the population.

Summary

Although distributional differences in lifespan are known to exist between population subgroups, how much these inequalities are contributing to the total lifespan inequality to date remains unknown. This study shows how lifespan inequality can be decomposed by subgroups, into its between- and within-group components. The example given relates to sex inequalities in lifespan, but the methods could equally be applied to quantifying the contribution of socioeconomic [15], racial, or regional lifespan inequalities to the total dispersion in age at death. Examining differences in distributions compliment can give a different and complimentary view to the large body of inequality research examining life expectancy differentials. Such studies will contribute to a better understanding and interpretation of the causes of total lifespan inequality, including the between-country variations in these inequalities. Moreover such a method can help identify the factors that are causing differences in lifespan variation.

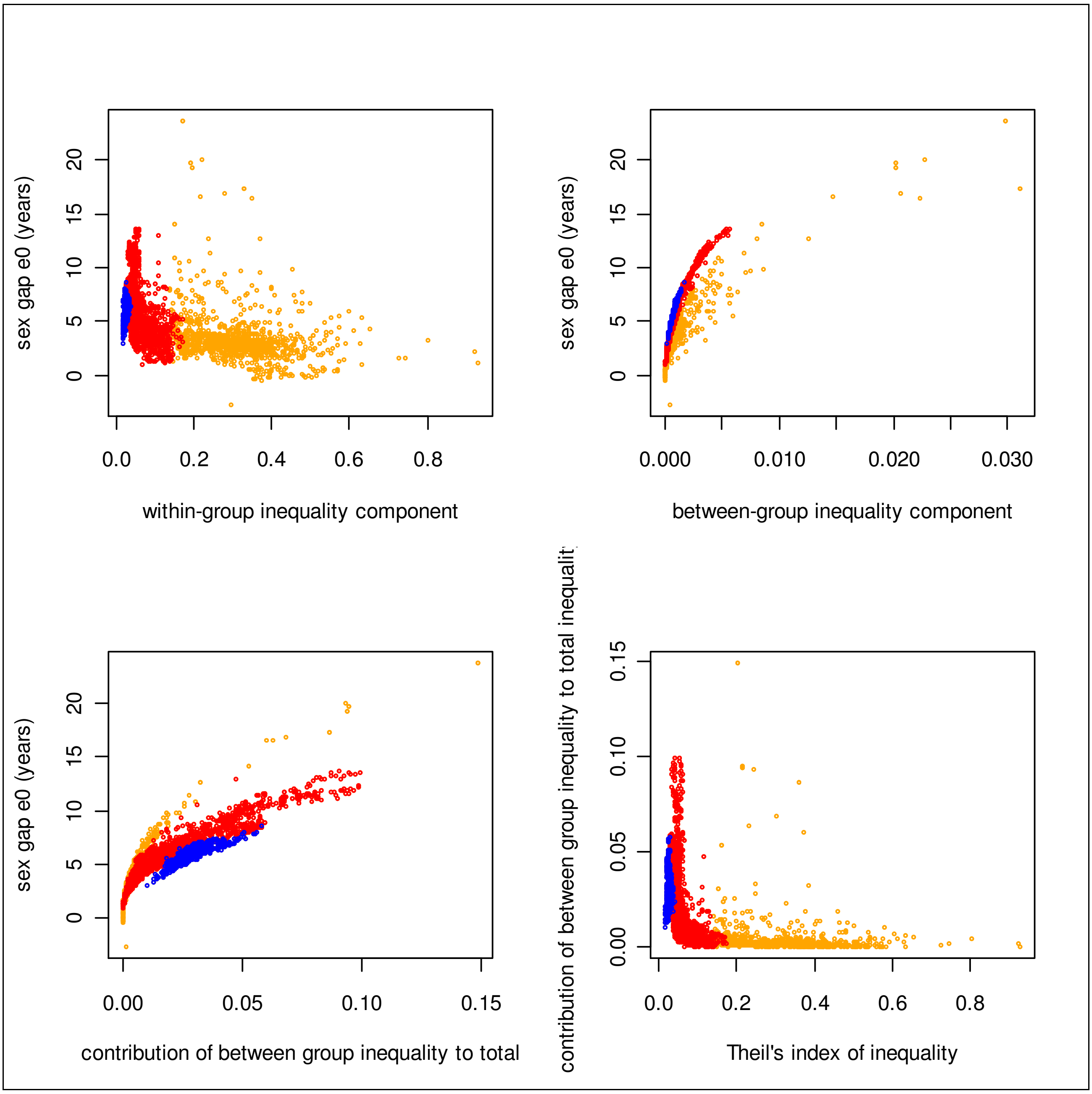

Figure 1: Results from the inequality decomposition for all years and countries of the Human Mortality Database. The panels show the association between the life expectancy gap between females and males and: the within-group inequality component (panel A), the between-group inequality component (panel B), and the proportion of total inequality explained by the between-group component (panel C). Panel D illustrates that this proportion was negligible at higher levels of total inequality. In all cases orange points refer to populations with a life expectancy below 60 years, red points to populations with life expectancies ranging between 60 and 75 years, while blue points are for populations with a life expectancy greater than 75 years.

Acknowledgements

I would like to thank Anton Kunst and Johan Mackenbach for their support and guidance in helping me to understand and develop these methods. I would also like to thank James Vaupel for his comments on an earlier draft.

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