Demographic Transition and Rapid Economic Growth: The Case of Taiwan.

Miguel Sánchez-Romero
Center on the Economics and Economics of Aging (CEDA)
*University of California, Berkeley**

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Abstract

The aim of this paper is to reconcile the debate between population and economic growth by considering the case in Taiwan. To solve this puzzle I compute a general equilibrium overlapping generations model with realistic demography. The main findings of the paper are twofold. First, the contribution of demography to economic growth during the demographic transition, is given by the difference between the growth rate of the number of employees and the population. Second, under a steady state equilibrium and assuming a stationary population, the economic growth rate relies on productivity; however, the economic level depends on the population age structure, where greater life expectancies and fertilities have a positive and negative effect, respectively.

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1 Introduction

Despite the general consensus that demographic factors influence saving rates and hence economic growth, the literature do not agree on the size of the effects. An economic model that can accurately estimate the effects of the age structure of the population on economic growth is of main interest for economies with both young and aging populations. The extraordinary economic experience of East Asia has provided a model for young populations. However, the aging population process urges most industrialized countries to have a better understanding of the connection between population growth and saving rates.

In the simplest life cycle saving model, children and retirees dissave, while workers save or accumulate capital, both to pay back the money borrowed when they were young and to finance their future consumption after they retire (Modigliani and Brumberg, 1954). Hence an economy with a high total dependency ratio (the ratio of youth plus the elderly to the working age population) should present low or negative saving rates and, conversely, an economy with a low total dependency ratio should have high saving rates. This relationship however is ambiguous in the empirical literature. Leff (1969), Modigliani (1970), Mason (1988), and Higgins and Williamson (1997) found a negative relationship between aggregate saving and total dependency rates. But, using different econometric techniques, dependency rates do not seem to be conclusive (Gersovitz, 1988).

There are several reasons that could explain the lack of a relationship between savings and population growth in many economic models. First and foremost, theoretical results are mainly based on stable population structures. Thus, the results from these models do not necessarily hold during the transition between two steady states. Second, children's consumption is in reality determined within the household and not directly by themselves, see Tobin (1967), Lee (1980). This implies that children neither save nor borrow. Instead, the working age population pays for the children's consumption by reducing their savings during childrearing and increasing them during their remaining lifespan. Third, following Bommier and Lee (2003), d'Albis (2007), and Lau (2007) the introduction of general mortality patterns into overlapping generations models (OLG) modifies the relationship between population growth and capital accumulation, which was previously obtained in models with unrealistic mortality patterns, e.g. Diamond (1965), Blanchard (1985), and Weil (1989). Fourth, elderly people receive financial support in a pay-as-you-go basis from their children and, in most industrialized countries, also from public institutions such as the social security. As a consequence, private savings are crowded out, reducing economic growth (Feldstein, 1974). Finally, the fact that annuity markets are not complete increases the likelihood of receiving bequest which also modifies the savings behavior.

In order to assess the contribution of demography on economic growth, in this paper I take into account all the enumerated reasons for a change in the savings behavior. I do so building a computational general equilibrium overlapping generations model of a closed economy. Then I use the model first to analyze how micro and macroeconomic variables of Taiwan evolve throughout the demographic transition. Second, I use the simulated results to study the convergence models. Among works that use this kind of model to quantitatively assess the economic consequences of demographic change are Auerbach and Kotlikoff (1987), Ríos-Rull (2001), Börsch-Supan et al. (2006), Chen et al. (2007), and Braun et al. (2009). The reason for focusing on the Taiwanese economy is twofold. First, Young (1995) shows that economic growth in Taiwan was mainly due to dramatic increases in input factors, rather than in the total factor productivity. The drastic change in the Taiwanese population during the last century thus provides a good framework for testing the relationship between population growth and capital accumulation. Furthermore, there exist several studies about the Taiwanese economy that attempt to respond to the same sort of questions and can be used as benchmark, including Deaton and Paxson (2000, 1997), Tsai et al. (2000), and Lee et al. (2000, 2001, 2003). Nevertheless, this paper differs from previous articles in that productivity factor prices are endogenously determined by the demographic structure of the population.

I arrive at two conclusions in this paper. First, the effect of the demographic transition through changes in bequest seems to have played an important role in the rapid economic growth of Taiwan during the period 1960 to 1990. Second, convergence models obtain the best results on the effect of demography on economic growth.

The remainder of the paper proceeds as follows: Section 2 introduces the model setup and the household problem with transfers. Section 3 is devoted to explaining the evolution of the savings behavior both under steady states and throughout the demographic transition. Section 4 compares per capita GNP with other demographic variables. Section 5 concludes. Finally, an Appendix containing the calibration, the techniques used to project the demography, and the programming of the model completes the paper.

2 The Model

The simulation results presented in this paper have been obtained computing a general equilibrium overlapping-generations model. Demographic projections of pseudo-Taiwan by one-year age and single-sex groups were estimated,[1] as well as a set of exogenous economic parameters, including labor-augmenting technological progress. Given this information set, the model computes path equilibrium prices for input factors (i.e. wages and interest rates), optimal consumption, investments, inter-vivos transfers, and unintentional bequests by age and time. Since the main interest of this paper is to study the impact of the demographic transition on economic growth, I analyze the period 1950-2050. Nonetheless, the model runs simulations from 1700 to 2250 for two reasons: first, to guarantee that the economy begins and ends up in a steady state equilibrium; and second, to avoid bias introduced by initial conditions.[2]

The economy is assumed to be closed to migration flows and capital investments from and to overseas. Therefore, the population increases through fertility and mortality and the stock of capital through the aggregation of the assets held by the population. Though this assumption will keep the simulation results away from the actual performance of the Taiwanese economy, this case is worth considering in order to understand how the demography impacts the economy, *ceteris paribus* other external shocks.

The economy is comprised of a competitive neoclassical firm, represented by a Cobb-Douglas production function, and of 101 representative individuals of each age-cohort from 0 to 100 with rational expectations. The number of people within each cohort will be denoted by the capital letter N. In general, the first subscript will be used for time and the second subscript for age. Every individual faces lifetime uncertainty that varies according to age and time. Let the probability that an individual survives from birth to age x in year t be $l_{t,x}$. All individuals regardless of when they are born start making decisions at age 21 (T_w) and retire at age 65 (T_r). In other words, children are completely dependent which implies that they cannot

[1]Pseudo-Taiwan differs from the actual Taiwanese population in that the former is a closed population. Similar population projections of Taiwan were previously done by Lee et al. (2000, 2001, 2003) to study the demographic transition and its macroeconomic consequences. For further details about how pseudo-Taiwan is modeled see Appendix B.

[2]Similar models assume the population is stable at the beginning of the period to analyze (Chen et al. (2007), Braun et al. (2009)). However, this assumption gives misleading results when one tries to separate out the effect of demography on savings, since they are not taking into account the fact that the actual population need several decades before it becomes stable.

have wealth or make any consumption decisions. Let us assume that there is no annuity market, so following Yaari (1965) individuals are borrowing constrained and leave at the time of death an unintentional bequest, or h. Rather than taking the unintentional bequest across all ages and spreading it out among those who are T_w years old, in this model surviving-offspring inherit their parents' wealth.[3] Let $o_{t,x}$ be defined as the number of adult surviving-offspring of an individual of age x in year t; or equivalently

$$o_{t,x} = \sum_{s=T_w}^{x} \frac{N_{t-x+s,s}\hat{f}_{t-x+s,s}}{N_{t,x}} l_{t,x-s} \cdot I_{x-s>T_w}, \tag{1}$$

where $\hat{f}_{t,x}$ is the age-specific fertility rate of an individual of age x in year t and I is an index function that takes the value of one when the inequality is satisfied and zero otherwise.[4] Then, the "expected" bequest by an individual at age x in year t is given by

$$\begin{aligned} h_{t,x} &= (1+r_t) \sum_{s=T_w}^{x} \frac{N_{t-x,s}\hat{f}_{t-x,s}}{N_{t-x,0}} \frac{N_{t,s+x}}{o_{t,s+x}} q_{t,s+x} a_{t,s+x} \\ &+ (1+r_t) \frac{q_{t,x}}{p_{t,x}} a_{t,x} I_{x<2\cdot T_w}, \end{aligned} \tag{2}$$

where r is the real interest rate, $q_{t,x}$ is the probability of dying between age x and $x+1$ among cohort members alive at age x, $p_{t,x}$ is the probability of surviving between age x and $x+1$ among cohort members alive at age x, and $a_{t,x}$ is the assets at age x in year t. The second term of Equation (2) takes account of the fact that individuals under 42 years old, or $2T_w$, cannot leave their stock of assets to their children. Equation (2) says that the amount of bequest to be inherited is positively related to the parent's wealth and the parent's mortality risk and negatively related to the number of siblings. Throughout the paper, I will show that bequest plays an important role when explaining the macroeconomic consequences of the demographic transition.

Parents pay for their children's consumption needs. The consumption of a child is assumed to be a fraction θ of the adult's consumption (measured in units of *Equivalent Adult Consumers*, hereinafter EAC). Thus, the childrearing cost for the household head not only depends on the number of

[3] The unintentional bequest given in the form of lump-sum to individuals at age T_w increases consumption at the expense of savings. As a consequence, savings are underestimated.

[4] The age-specific fertility rate is truncated so that it matches with the assumption that children do not make any decision. For more detail, see Appendix B.

surviving children but also on their ages. Let the number of EAC within a household whose head is x years old in year t be $\lambda_{t,x}$. Thus,

$$\lambda_{t,x} = 1 + \sum_{s=T_w}^{x} \theta_{x-s} \frac{l_{t-x+s,s}}{l_{t,x}} l_{t,x-s} \hat{f}_{t-x+s,s} I_{x-z<T_w}. \tag{3}$$

Elderly parents also receive financial support from their offspring, which will be called old-age support (OAS). Specifically, the amount of money transferred to each elder person is equal to 40% (π^{oas}) of the average labor income of her offspring. This transfer scheme to old people implies that the cost per offspring is negatively correlated with the number of siblings, similar to an unfunded social security system. This is an important assumption, because in Taiwan the majority of elderly people live with their offspring and it has one of the highest elderly support expenditures per family member, see Lee et al. (1994) and Deaton and Paxson (2000). The fraction of labor income of an individual of age x in year t allocated to financially support her elder parent is

$$\bar{\tau}^{oas}_{t,x} = \pi^{oas} \sum_{s=T_w}^{\Omega-1} \frac{N_{t-x,s} \hat{f}_{t-x,s}}{N_{t,0}} \frac{N_{t,s+x}}{o_{t,s+x}} I_{s+x \geq T_r}. \tag{4}$$

According to Equation (4) the cost of supporting the elderly for any cohort is a function of the number of siblings. Thus, OAS is expected to be higher for the baby-bust generation and lower for a baby-boom generation.

In short, individuals give or receive transfers all along their life span. First, parents pay for individuals' consumption needs when they are young. Second, during their working period individuals transfer money to financially support their children and their elderly parents. Third, they receive support from their adult-children when they are old. Fourth, at all ages, individuals "expect" to receive an unintentional bequest from their parents. Nevertheless, the existence of transfers does not mean that these transfers are fixed over time. Indeed, transfers not only will change because of optimal economic decisions made by all living individuals, but also because of changes in the age structure of the population and the composition of the household.

2.1 The household problem

The optimal allocation of resources across the life cycle is modeled using an extended version of the life cycle theory of saving proposed by Tobin (1967).

In particular, the household problem with transfers presented in this paper relies on the work previously done by Lee et al. (2001).

All individuals are forward-looking and do not have a bequest motive. Therefore, every cohort tries to consume the stream of labor income and net familial transfers (inflows and outflows). Individuals leave their parent's house at age T_w and form their own household. Up to the age of retirement T_r, they supply their labor inelastically to a neoclassical firm in exchange for a salary. Let $y_{t,x}$ be the salary earned by an individual of age x in year t, which depends on an age-specific labor productivity index (ϵ_x) that does not vary over time, on the salary in units of effective labor in year t, or w_t, and on the labor-augmenting technological progress, or A.[5]

$$y_{t,x} = w_t A_t \epsilon_x \text{ with } T_w \leq x < T_r. \tag{5}$$

Individuals also receive both an accidental bequest when their parents die and financial support from their children when they are retired. Similarly, they finance the consumption of their children (childrearing cost) while they are in the household and support their parents when the latter retire. It is assumed that all individuals are risk averse and have the same preferences and tastes along their uncertain life span, which are represented by a time-separable constant relative risk aversion (CRRA) utility function v. Then, the optimal consumption along the life-cycle for the cohort born in year t is given by the following maximization problem:

$$\max_{\{c_{t+x,x}\geq 0\}_{x=T_w}^{\Omega-1}} v(t+T_w, T_w) = \sum_{x=T_w}^{\Omega-1} \beta^{x-T_w} \frac{l_{t+x,x}}{l_{t+T_w,T_w}} \lambda_{t+x,x} u(c_{t+x,x}), \tag{6}$$

subject to an age-dependent flow budget constraint, which is equal to

$$s_{t+x,x} = r_{t+x} a_{t+x,x} + h_{t+x,x} + (1-\tau^{oas}_{t+x,x}) y_{t+x,x} - \lambda_{t+x,x} c_{t+x,x},$$

when the individual is in the labor market, and

$$s_{t+x,x} = r_{t+x} a_{t+x,x} + h_{t+x,x} + \pi^{oas} \bar{y}_{t+x,x} - \lambda_{t+x,x} c_{t+x,x}, \tag{7}$$

when the individual is retired. Furthermore, given that there is no annuity market, individuals cannot borrow money, so that $a_{t,x} \geq 0$ always. Where $\beta \in (0, 1]$ is the subjective discount factor, s is private savings, a is the number of assets, λc is the household consumption, and $\bar{y}$ is the average

[5]See Appendix C for further detail on how the age-specific labor productivity index is derived.

labor income of the offspring. Like Equation (1), the average labor income $\bar{y}$ is equal to

$$\bar{y}_{t,x} = \sum_{s=T_w}^{x} \frac{N_{t-x+s,s}}{N_{t,x}} \frac{y_{t,x-s}}{o_{t,x-s}} l_{t,x-s} \hat{f}_{t-x+s,s} \cdot I_{x-s>T_w}. \quad (8)$$

An important feature of the expected utility (6) is that the slope of the household head's consumption is independent of the size of the household. Therefore, transfers just affect the level of the age-consumption profile and not to a particular age. Thus, using (6) and (7), the optimal consumption path for this problem is:

$$\beta p_{t+x,x}(1 + r_{t+x+1})u'(c_{t+x+1,x+1}) \geq u'(c_{t+x,x}), \quad (9)$$

with equality whenever $a_{t+x+1,x+1} > 0$. Now, given that household heads smooth their consumption, savings will vary according to the size of the transfers made and received.

3 Savings behavior

In reality, individuals rear their children, support their parents directly through familial transfers or indirectly through payroll taxes, and face borrowing constraints. These transfers move saving decisions actually undertaken by individuals away from the optimal saving decisions predicted by the simple life-cycle model. Why? because individuals smooth their consumption, and so greater or lower temporary transfers to children and from parents mostly affect savings. While permanent and constant transfers equally affect consumption and savings. How familial transfers are related to age and time, individuals' savings are conditioned by the stage of the demographic transition in which they are living. Changes in life expectancies, number of children, and the population age structure have a strong impact on household's economic behavior, to the extend that we can observe important variations at micro and macroeconomic levels along the demographic transition.

3.1 Steady-state equilibrium

Based on the outcomes of the model, it is observed that before the demographic transition starts (high mortality and fertility rates), the consumption of the household head follows the simple life-cycle model described by

Modigliani and Brumberg (1954). Figure 1 below shows simulated outcomes for income, consumption, transfers, and savings. Adults consume more than they earn in the labor market while young, consume less than their wage during their prime working ages, and dissave when they retire. However, this does not imply that they are actually saving in their prime ages nor that they are faced with borrowing constraints. On the contrary, individuals save from age 20 to age 50, dissave from ages 50 to 80, and they are liquidity constrained afterwards. This age profile of saving is repeatedly observed even for different productivities. How is this possible? This is simply because of the existence of transfers. The non-existence of a complete annuity market coupled with the fact that the life expectancy at birth is low (with a high variability) stimulate individuals to accumulate more assets.[6] Indeed, with an annuity market individuals will not receive bequest and they will borrow money in order to have a similar age-consumption profile than that in Figure 1. Fertility is also high, thus they have to save in advance before the childrearing cost will peak in their 40s. These two saving forces driven by the demography lead the individual to be able to save when young and reach her maximum wealth at age 45. At the same time, the probability of dying at 40 years old is also very high, and so it is likely that assets are accidentally transferred in the form of bequest to their children. The expected unintentional bequest is 40 percent of the household head's consumption at age 21 and slowly declines up to 13 percent at age 40. This bequest is partly consumed by the head of the household and partly saved in order to finance both the cost of childrearing and, to a lesser extent, her own consumption after retirement. After the demographic transition bequests will be lower and are received later in life.

Given these demographic characteristics and transfers, it is worth noting that this allocation process maximizes individuals' welfare since delaying the accumulation of assets by raising consumption in the age-range 20 to 30 prevents the economy from reaching a higher welfare. For example, lower savings at young ages cause an overall reduction of both wealth and bequest, which ultimately implies that young workers will face borrowing constraints that reduce their welfare. Then, the optimal saving behavior when the risk of mortality is high is to accumulate a disproportionate amount of assets. Paradoxically, these assets will probably finance not her own consumption, but the consumption of her children.

[Figure 1 about here.]

[6] This variability can only be proved with realistic survival probabilities.

At the macroeconomic level, since the population age structure is tilted towards young ages, the "pre-transition" household saving pattern implies that national net savings will be positive and close to 12% for a total factor productivity (TFP) growth rate of 1.5%, and above 15% for a TFP growth rate of 0%. The negative correlation between national savings and TFP growth is due to the existence of transfers. The lower TFP growth implies that capital input is diminished. Hence, the lower interest rate increases consumption while young and decreases it at older ages, which might cause us to expect a reduction in savings. Nevertheless, individuals now need more assets than before in order to have the same asset income at prime ages to finance the cost of childrearing. As a result, this greater wealth at age 40 is transferred in the form of bequests to the children, which allows them to increase their savings.

After the demographic transition when mortality risk and fertility are low, the consumption of the head of the household is flatter across her life span, smaller relative to her labor income, and peaks when she is 80 years old. This old-age consumption pattern was described by Hurd (1989) using the Retirement History Survey (RHS). Consumption relative to her average labor income from 30 to 50 years old is the result of the combination of four effects. On the one hand, overall consumption decreases because the higher life expectancy at age 21 dominates the desire to increase the consumption in the present (Levhari and Mirman (1977)). Also, the longer life expectancy after retirement and the fact that there are less siblings increase the cost of supporting elderly parents, over 25% of their labor income among ages 36 to 57, which reduces consumption and total income at prime ages (relative to the average labor income). On the other hand, the consumption of the household head increases at younger ages because the real interest rate is lower after the demographic transition and because she does not spend as much in childrearing due to the lower fertility. The former effects dominate the latter effects and hence consumption relative to the average labor income from 30 to 50 years old decreases.

Individuals change their savings because the new age distribution of the population modifies the amount of money transferred. At the very beginning of adulthood, individuals are borrowing constraint since they do not receive bequests. Now, elders allocate better their resources along their remaining life span owing to the lower variability in their survival probabilities. Thus, the unintentional bequest is lower across all ages.[7] Individuals save from

[7] The evolution of the unintentional bequest presented here is consistent with the empirical finding of Kotlikoff and Summer (1981) and subsequent articles analyzing the size

ages 25 to 56, dissave before and during retirement, and their consumption equals the old age support received from their adult children at the end of their life spans.

There are two important changes in the post-demographic transition saving pattern relative to that of the pre-demographic transition. First, the longer life expectancy boosts savings for retirement from 50 to 65 years old. Second, after the demographic transition, net transfers during the working life become greater than before the demographic transition and are always negative.[8]

The national net savings rate in the post-demographic transition is close to 10% per year, which is lower than the pre-demographic transition savings rate. Two reasons explain the reduction in savings. First, the aging population leads to the second demographic dividend. Labors have more capital available but they are less. Consequently, production in units of effective labor grows less rapidly than capital per unit of effective labor decreasing the real interest rate (2.5% lower than pre-transition real interest rate), which is a deterrent to save and invest. Second, the age-specific labor productivity index profile and the old-age support cost constrains savings.[9] Therefore, the stock of capital is lower than it could be. In order to stimulate higher savings rates, it would be necessary to raise productivity levels or decrease the financial support of the adult children to their parents.[10]

[Figure 2 about here.]

These results do not imply that young populations will save more relative to aging populations. Transfers and the lack of complete annuity markets are the cause for the different outcomes of this paper. Indeed, when I remove the old-age support cost and model the economy with annuities, savings in aging populations are greater than in young populations. Therefore, given that the tandem transfers and demography can account for the change in

and the degree to which unintentional bequest accounts for the current accumulation of capital in the U.S., e.g. Abel (1985), Hurd (1989), and Gokhale et al. (2001).

[8]This is true under the assumption that, regardless of the number of offspring, the financial support of elderly people is kept constant.

[9]Following Feldstein (1974) this is due to the crowding-out effect that the old-age support cost imposed to the system. Note that the OAS is modeled as a familial pay-as-you-go system.

[10]Other alternative could be the increase of the labor participation rate. This policy however can yield either an increase or a decrease in the savings rate. According to simulations not presented here, an increase of the participation rate before the mandatory age of retirement increases savings, but a postponement of the age of retirement would probably have the opposite effect.

consumption and savings, I think that it is important to estimate the extent to which transfers are relevant in an economy in order to analyze the relation of demographic change to economic growth.

3.2 Demographic transition

The demographic transition is the change from high fertility and mortality rates to low fertility and mortality rates. This change modifies not only the age-structure of the population but also relative prices of inputs and transfers. The most remarkable and recognizable economic impact happens when the population growth rate is maximum (i.e. high fertility and low mortality), but important changes take place since the first stage of the demographic transition; i.e. when fertility rates are high and mortality rates start to decline. The decline in mortality rates shifted the level of consumption downwards and made consumption smoother, so savings tilted towards young ages. In addition, population began to increase more rapidly due to lower mortality. Hence, net national savings rise at the expense of the consumption of young workers. Interest rates decreased and salaries in units of effective labor increased, providing a greater welfare. However, this effect is counterbalanced by the decrease in child mortality and lower bequest.

During the first half of the twentieth century total fertility rates (TFR) increased from 6 to 7. This change would not heavily impact the economy if child mortality had remained constant, however this was not the case. Population started a rapid rate of increase that peaked in 1950 at over 3 per cent per year. The greater welfare caused by the slight decrease in mortality after age 20 was turned down because of the raise in the TFR and the improvement in child mortality. Childrearing cost increased sharply reducing savings. Interest rates backed up because of the reduction in savings and because of the lower capital to labor ratio imposed by the increase in the labor supply.

Without an initial stock of capital or a very low interest rate imposed from overseas, the model suggests that Taiwan would have experienced a sharp decline in production per capita. The decline of infant mortality and high fertility rates led young individuals to be unable to save. This was partly balanced by the fact that survival probabilities around 1950 were low, and so young generations still received accidental bequest. At the same time, firms demanded more capital in order to maintain their capital-to-labor ratio, which yielded an increase in interest rates and savings. Therefore, it can be said that the importance of transfers coupled with the increasing labor force supply led to high savings rates regardless of the productivity level.

But, how is it possible that savings rates were also extremely low in 1953? The reason seems to be the investments in education. Following Huang (2001) the proportion of illiterate and self-educated people by birth cohorts from 1925 to 1940 was between 13-16% for males and between 37-50% for females. In 1944 the constitution of the Republic of China established six years of compulsory primary education and it was extended again in 1968 to nine years. The proportion of illiterate and self-educated people was then progressively reduced almost to 0% in 1970. For this reason, it is likely that the investment in education played an important role raising the productivity in Taiwan from 1960 to 1990. Hence, the expectation of higher future productivities increased consumption before 1950 at the expense of lower savings.[11] To see this point Figure 3 shows actual and simulated private savings under two scenarios. The astonishing increase in private saving rates from 1953 to early 70s (actual savings and Simulation 2) is explained by both the increase in the labor supply and higher productivities. For example, under constant productivity levels of 1.5% (Simulation 1) just the increase in labor supply yields an increase of almost 1 percent in the real interest rate (from 6% to 7%). While the higher productivity of Simulation 2 increases the real interest rate from 6% to 12% per year, boosting savings more rapidly in Simulation 2 than in the former simulation. This period corresponds to the demographic bonus first presented by Higgins and Williamson (1997). That is, the period in the demographic transition where the number of producers increases more rapidly than the number of consumers.

[Figure 3 about here.]

Under constant productivity growth rates, Simulation 1, we see that savings drop after 1980. The entrance of the baby-boom generation into the labor market lead the youth dependency ratio and interest rates to decline.[12] The stable saving rates from 1960 to 1980 is caused by two opposite effects. First, as the youth dependency ratio decreases, individuals have more disposable income that increase savings. This effect is however balanced with lower interest rates, which increase consumption at the expense of savings.

[11] Another plausible explanation, which cannot be analyzed within this model, is that the education cost per capita rose not only because of the higher investment but also because of the high child dependency ratio. Thus, net savings during the 40s and 50s were small because the education burden was supported by the living population at the expense of their savings.

[12] See Figure 8 in Appendix B.

Similarly, the subsequent decline in savings after 1980 is due to two reasons: the decline of interest rates and population aging.

Comparing simulations 1 and 2, we can see that saving rates up to 22% were driven by the population age structure and that any difference from this value was due to changes in the labor-augmenting technological progress. Also, it is worth noting that productivity played a different role before and after 1970. This is probably due to the change in the bequests received. Thus, higher productivities before 1970 pushed down savings, while after 1970 shifted them upwards. This is true for all simulations run with very different labor-augmenting technological progresses ranging from 0% up to 5.5%.

[Figure 4 about here.]

The last stage of the demographic transition is characterized by population aging. On the one hand, savings decrease because the mean age of saving approaches to age 45, where both the childrearing cost and old-age support cost are maximum. On the other hand, the probability of inheriting bequests from their parents becomes less likely given that the uncertainty of the age of death is smaller. The savings floor in 2050 is produced by the imbalance between the number of workers relative to the number of elderly. In this particular case, the old-age support cost per offspring will progressively rise up to doubling the π^{oas} (i.e. 42%), taking over the position that childrearing cost had during the baby-boom period. Nonetheless, this percentage can be faced by future workers because of the increase the capital-to-output ratio, or second demographic dividend, as Figure 4 shows. Similar results were previously obtained by Lee et al. (2000, 2001, 2003).

To sum up, the demographic transition seems to have played an important role in the rapid economic growth of Taiwan during the period 1960-1990, otherwise, the outcomes obtained here would have not matched the actual savings rates. The productivity shock from 1960 to 1990 accounts for the rest of the rapid growth. The next section will be devoted to explaining how the macroeconomic performance through this period, except for savings, can also be explained by the demography.

4 Convergence models: macroeconomic performance.

The information that this computational equilibrium overlapping-generations model provides can give new insights into the outcomes of the model developed by Bloom and Williamson (1998) in page 431. In particular, in this section I am interested in testing the comment written by Williamson (2003):

> *"We start by asking whether the level of population growth affects economic growth, since that's the* **(wrong)** *way the population debate has always been couched."*(Williamson, 2003, p. 114)

Since this model is based on several unrealistic assumptions, such as the non-existence of unemployment, and closed economy without migration flows, among others, this task can be done if, and only if, Simulation 2 fits reasonably well to actual economic data. If this is true, we can then study the convergence model focusing on Simulation 1 since it is not affected by changes in productivity. In other words, we will be able to analyze how the demographic transition impacts GNP per capita, controlling for the effect of productivity.

Table 1 compares average annual growth rates of actual macroeconomic data with those obtained from simulations 1 and 2 for the period 1961-2003. The most significant difference is the underestimation of the growth rates of the variables per capita *GNP* and *Capital Stock*. This is because the labor-augmenting technological progress in Taiwan was, on average, greater than 1.5% per year, see Figure 7. Difference between actual data and those obtained from Simulation 2 are not signigicant. Nonetheless, it is worth highlighting that the underestimation of the growth rate of the variable per capita *GNP* for periods 1961-1979 and 1990-1999 is consistent with the underestimation of the difference between labor and population growth rates. This is probably because I am modeling an economy closed to migration flows. In fact, immigrants from Mainland in the 1950s with different fertility and mortality rates might explain the gap.

Table 1: The macroeconomic performance in Taiwan (average annual growth rate, %)

Year	Per capita GNP	Capital Stock	Employment (L)	Population (N)	Difference L-N
Actual Data					
1961-1969	6.21	4.87	3.00	2.37	0.63
1970-1979	7.14	9.44	3.42	1.80	1.62
1980-1989	6.78	7.93	2.38	1.25	1.13
1990-1999	4.89	8.49	1.39	0.81	0.57
2000-2003	1.52	4.28	0.74	0.51	0.23
Simulation 2					
1961-1969	5.74	4.92	3.15	2.56	0.59
1970-1979	6.45	8.85	3.28	1.89	1.39
1980-1989	6.77	7.84	2.91	1.43	1.48
1990-1999	4.72	8.02	1.91	1.05	0.86
2000-2003	1.62	4.59	1.07	0.54	0.53
Simulation 1					
1961-1969	1.96	4.72	3.15	2.56	0.59
1970-1979	2.70	4.75	3.28	1.89	1.39
1980-1989	2.84	4.41	2.91	1.43	1.48
1990-1999	2.22	3.68	1.91	1.05	0.86
2000-2003	1.54	2.61	1.07	0.54	0.53

SOURCE: Time series from 1951 to 2004 of the National Accounts of the Republic of China collected from CEIC Global Database and Author's calculations. NOTES: In order to avoid bias, all average values are calculated through geometric means. In the simulations the variable "Employment" is calculated as the rate of growth of the active population.

The fundamental idea that underlies the convergence model is that through the demographic transition the growth rate of GNP per capita is explained by the difference between labor and population growth rates $grL - grN$ and not by any of these variables independently. The simulated results point in the same direction. Using the results from Simulation 1 (constant productivity) plotted in Figure 5 we can see that neither N nor L can be used to explain the growth rate of the GNP per capita. First, because the relationship is not linear, and second, because the effect (direction and intensity) of the demographic transition on GNP per capita varies over time. Note that the relationship between GNP_{pc} and the population growth rate grN from 1950 to 1959 is ambiguous, negative from 1960 to 1979, probably negative but not significant from 1980 to 1989, positive from 1990 to 1999, and so on. Like the variable grN, the relationship between GNP_{pc} and the growth rate of the economically active population grL gives similar results, see Figure 5 at the bottom. Moreover, consistent with Kelley and Schmidt (2003), the net effect of both variables on GNP per capita is zero as we approach to a stable population.

[Figure 5 about here.]

As was expected, convergence models predict the observed data very well as Figure 6 shows. That is, population growth has positive and negative effects. During both the baby boom and population aging, the growth rate of GNP will be lower than its potential growth because of the increase in the number of dependents. On the contrary, the first demographic dividend increases GNP per capita. Thus, we observe that the higher the difference between the economically active population and the population growth rate is, the higher the growth rate of per capita GNP becomes. Unfortunately, looking at Figure 6 several problems come up in a simple regression analysis. First, time series that mostly contain information of the first demographic dividend can give misleading results. For example, a linear regression of those points located where the first demographic dividend is maximum (i.e. $grL - grN$ is over 1.5%) gives a zero slope. Second, changes in the education level, labor force participation rates, and transfers modify the impact that demography has on the economy. Otherwise, subfigure (a) in Figure 6 should have a slope equal to one. Third, variations in productivity masks the real effects of demographic changes, so that productivity levels need to be controlled.

[Figure 6 about here.]

5 Conclusion and caveats

In this paper I have measured the contribution of demography for economic growth. To do so, I have used a general equilibrium overlapping generations model with realistic demography and transfers, such as childrearing cost, old-age support, and accidental bequests.

I have arrived at two conclusions in this paper. First, the effect of the demographic transition through changes in the amount of bequests seems to have played an important role in the rapid economic growth of Taiwan during the period 1960 to 1990. Second, convergence models obtain good results on the effect of demography on economic growth.

As with any other model, there are several features that can be improved in future research. First, most economies are not closed to migration flows nor to international capital markets. Therefore, a model that aims to forecast future economic growth trends, and not just the effect of demography, should extend the model by considering an open economy. I have not considered here investments in human capital nor different age-specific labor productivity indexes. Undoubtedly, changes in education are going to have an important impact on future savings, and thus they should be introduced. The results presented here, however, are not affected much because I have used income data from the National Transfer Accounts from 1978 to 2003. Finally, this model does not contemplate unemployment, which is another important cause for changes in savings behavior.

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A Calibration

In the model economy the value of γ is set to 0.60, which is the average value estimated for Taiwan by Reinhart et al. (1996). Usual values for this parameter range between 0.5 and 1, so that γ is within the range frequently used in the literature. Age-specific labor productivity indexes are calculated using data from the National Transfers Account Database (see Appendix C). The subjective discount factor is set at 0.99. With National Accounts of the Republic of China from CEIC Global Database I calculated the stock of capital for Taiwan by using the time series of "Gross Fixed Capital Formation" (GFCF) and the "Consumption of Fixed Capital" from 1953 to 2004. I discarded the information from 1953 to 1960. The average value of the depreciation of capital (δ) from 1975 to 2003 that I obtained was 3.25%. Based on a Cobb-Douglas production function with constant returns to scale and under the assumption that the economy is closed, the value of capital share ($\alpha = 0.32$) was estimated as the average value from 1951 to 2004 as follows:

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T} \frac{\mathrm{O.S}_t - \mathrm{E.I}_t}{\mathrm{N.I}_t - \mathrm{E.I}_t - \mathrm{N.I.T}_t}, \tag{10}$$

where O.S. is "Operating Surplus", E.I. is "Entrepreneurial Income", N.I. is "National Income", and N.I.T. is "Net Indirect Taxes".

Table 2: Parameters in the OLG Model

Parameters	values
β: subjective time discount factor	0.99
$\frac{1}{\gamma}$: relative risk aversion	1.67
δ: capital depreciation rate	3.25%
α: capital share	0.32
T_w: first age at the labor market	21
T_r: age of retirement	65
Ω: maximum longevity	100

Labor-augmenting technological progress (A) was calculated using the formula:

$$\frac{GNP_t}{N_t} = \hat{A} \cdot \left(\frac{K_t}{GNP_t} \right)^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} \frac{L_t}{N_t}. \tag{11}$$

Where GNP is Gross National Product and $\frac{K_t}{GNP_t}$ is the capital-to-output ratio. The initial value of the capital-to-output ratio was set at 2.7 in 1960. Figure 7 below shows the results.

[Figure 7 about here.]

B Baseline Demography and Tables

In this section of the appendix I explain the procedure I followed to model the demographic projection. Before I start the explanation it is important to highlight that I am interested in the period from 1950 to 2050, however the population projection will start in 1721 and end up in 2250. Obviously, my first intention is not to have an accurate demography before and after the period 1950-2050, but to provide enough time so that the population will be stable. In other words, the population will grow at a constant rate. This is a necessary economic assumption imposed on the demography for the economy to settle down and reach a steady state equilibrium.

Following Lee et al. (2000, 2001, 2003) the population modeled is pseudo-Taiwan. The population is named pseudo-Taiwan because it is based on projections of mortality and fertility from historical data, see (Lee et al., 2000, p. 202). Thereby, pseudo-Taiwan population differs from the actual Taiwanese population in that the former is a closed population (without migration).

First, I use single-sex period-life expectancies at birth (e_0) and period-total fertility rates (TFR) from Lee et al. (2000, 2001, 2003). Age-specific fertility rates from 1951 to 2006 were collected from the *Statistical Year Book of the Republic of China* 1975, 2008. Age-specific mortality rates from 1951 to 2005 come from the *Statistical Year Book of the Republic of China* 1975 and from the Human Mortality Database. Second, all the data collected from the Statistical Year Books of the Republic of China were transformed from five-years age groups to single-year age groups by interpolating splines. Third, age-specific mortality rates for ages 85 and over were from 1951 to 1970 in an open interval. Thus, for the sake of consistency with Human Mortality Database, I used the Kannisto model to extrapolate the data up to age +110. Table 3 below summarizes the estimated parameters by year using a non-linear least square regression.

Table 3: Kannisto Model: Estimated Parameters for Taiwan 1951-1970.†

Year	$\hat{a}$	t-student$_{\hat{a}}$	$\hat{b}$	t-student$_{\hat{b}}$
1951	.0130 (.0059)	2.1937	.0746 (.0377)	1.9792
1952	.0105 (.0048)	2.1850	.0804 (.0379)	2.1245
1953	.0105 (.0048)	2.1850	.0826 (.0378)	2.1870
1954	.0102 (.0047)	2.1890	.0801 (.0378)	2.1192
1955	.0097 (.0044)	2.1871	.0862 (.0378)	2.2822
1956	.0095 (.0043)	2.1977	.0871 (.0376)	2.3205
1957	.0094 (.0043)	2.2031	.0896 (.0374)	2.3948
1958	.0088 (.0040)	2.1911	.0873 (.0377)	2.3160
1959	.0083 (.0038)	2.1792	.0909 (.0380)	2.3947
1960	.0083 (.0038)	2.1896	.0906 (.0377)	2.4016
1961	.0083 (.0038)	2.1753	.0888 (.0381)	2.3331
1962	.0078 (.0036)	2.1809	.0919 (.0379)	2.4216
1963	.0073 (.0034)	2.1688	.0945 (.0382)	2.4743
1964	.0072 (.0033)	2.1716	.0935 (.0381)	2.4515
1965	.0071 (.0033)	2.1672	.0924 (.0383)	2.4149
1966	.0068 (.0031)	2.1846	.0966 (.0378)	2.5528
1967	.0069 (.0031)	2.1942	.0951 (.0376)	2.5263
1968	.0071 (.0032)	2.1961	.0945 (.0376)	2.5137
1969	.0067 (.0030)	2.2015	.0939 (.0375)	2.5039
1970	.0063 (.0029)	2.1951	.0979 (.0376)	2.6017

Note: Standard deviations in parenthesis.

†Parameters {a,b} come from the hazard rate $\mu(x) = \frac{ae^{b(x-x_0)}}{1+ae^{b(x-x_0)}}$.

Fourth, matrices of age-specific fertility rates and mortality rates were used to calculate the age-specific (spines) constants $\mathbf{a}_x$ and $\mathbf{b}_x$ of the Lee-Carter Model for fertility and mortality, respectively. Following Lee and Carter (1992) and Lee (1993) I assumed that fertility is an additive process:

$$\mathbf{f}(t,x) = \mathbf{a}_x^f + \mathbf{f}_t \cdot \mathbf{b}_x^f + \epsilon_{t,x}^f, \text{ where } \epsilon_{t,x}^f \sim i.i.d.(0, \sigma_{\epsilon^f}^2), \quad (12)$$

while mortality is a multiplicative process:

$$\log \mathbf{m}(t,x) = \mathbf{a}_x^m + \mathbf{k}_t \cdot \mathbf{b}_x^m + \epsilon_{t,x}^m, \text{ where } \epsilon_{t,x}^m \sim i.i.d.(0, \sigma_{\epsilon^m}^2). \quad (13)$$

The results are reported in tables 4 and 5 below

Table 4: Taiwan: Age-Specific Fertility Rates (1951-2006), Lee-Carter Model.

Age	a_x^f	b_x^f	Age	a_x^f	b_x^f
13	1.4835	-1.9841	33	119.7496	-222.5855
14	5.6054	-7.4845	34	105.2794	-217.5134
15	11.8731	-15.8232	35	91.3772	-211.3093
16	19.7939	-26.3222	36	78.8489	-202.6574
17	28.8750	-38.3035	37	68.5000	-190.2421
18	46.7427	-62.1450	38	59.4455	-173.9753
19	76.5151	-101.7373	39	50.5136	-155.4223
20	110.9376	-146.6717	40	42.0805	-135.7473
21	142.7555	-186.5391	41	34.5221	-116.1144
22	164.7143	-210.9310	42	28.2143	-97.6880
23	179.7073	-222.5979	43	22.5546	-79.5992
24	194.3302	-231.3525	44	16.9901	-61.2746
25	206.8815	-237.3783	45	11.9716	-44.3220
26	215.6600	-240.8590	46	7.9496	-30.3490
27	218.9643	-241.9785	47	5.3750	-20.9632
28	212.3414	-241.4340	48	3.7295	-14.5968
29	195.6962	-239.7795	49	2.2644	-8.8933
30	173.8644	-236.9835	50	1.0846	-4.2767
31	151.6812	-233.0146	51	0.2949	-1.1709
32	133.9821	-227.8412			
Frobenius Norm					
$1 - \frac{\|\|e'e\|\|_F}{\|\|y'y\|\|_F}$	.7253				

Note: Data in . Author's calculus.

Table 5: Taiwan: Age-Specific Mortality Rates (1951-2005), Lee-Carter Model.

Age	a_x^m	b_x^m	Age	a_x^m	b_x^m	Age	a_x^m	b_x^m
0	-4.3930	-0.1665	36	-6.0221	-0.0756	72	-3.0667	-0.0553
1	-5.5943	-0.3100	37	-5.9563	-0.0739	73	-2.9710	-0.0524
2	-6.1321	-0.2979	38	-5.8896	-0.0719	74	-2.8756	-0.0495
3	-6.5075	-0.2907	39	-5.8145	-0.0711	75	-2.7828	-0.0466
4	-6.8097	-0.2799	40	-5.7542	-0.0718	76	-2.6887	-0.0440
5	-7.0725	-0.2616	41	-5.6834	-0.0705	77	-2.5959	-0.0410
6	-7.2871	-0.2203	42	-5.5990	-0.0669	78	-2.5029	-0.0384
7	-7.4670	-0.1903	43	-5.5354	-0.0669	79	-2.4038	-0.0347
8	-7.5565	-0.1736	44	-5.4661	-0.0675	80	-2.3497	-0.0348
9	-7.6652	-0.1678	45	-5.3931	-0.0663	81	-2.2624	-0.0330
10	-7.7072	-0.1525	46	-5.3137	-0.0652	82	-2.1747	-0.0298
11	-7.7329	-0.1417	47	-5.2406	-0.0651	83	-2.0914	-0.0278
12	-7.6944	-0.1331	48	-5.1648	-0.0658	84	-2.0065	-0.0249
13	-7.6017	-0.1247	49	-5.0822	-0.0652	85	-1.9301	-0.0234
14	-7.4729	-0.1145	50	-5.0070	-0.0669	86	-1.8424	-0.0210
15	-7.2249	-0.0807	51	-4.9192	-0.0659	87	-1.7623	-0.0190
16	-7.0359	-0.0671	52	-4.8410	-0.0675	88	-1.6868	-0.0162
17	-6.9156	-0.0619	53	-4.7589	-0.0677	89	-1.6101	-0.0159
18	-6.7592	-0.0470	54	-4.6768	-0.0684	90	-1.5421	-0.0149
19	-6.7421	-0.0620	55	-4.5902	-0.0677	91	-1.4661	-0.0121
20	-6.7488	-0.0825	56	-4.5096	-0.0689	92	-1.3959	-0.0115
21	-6.7086	-0.0866	57	-4.4281	-0.0694	93	-1.3227	-0.0091
22	-6.6335	-0.0863	58	-4.3445	-0.0701	94	-1.2502	-0.0069
23	-6.5798	-0.0821	59	-4.2539	-0.0714	95	-1.1817	-0.0050
24	-6.5564	-0.0833	60	-4.1585	-0.0708	96	-1.1158	-0.0033
25	-6.5483	-0.0856	61	-4.0750	-0.0727	97	-1.0519	-0.0017
26	-6.5182	-0.0833	62	-3.9782	-0.0723	98	-0.9901	-0.0001
27	-6.4986	-0.0843	63	-3.8920	-0.0723	99	-0.9305	0.0014
28	-6.4564	-0.0840	64	-3.7983	-0.0717	100	-0.8732	0.0028
29	-6.4206	-0.0833	65	-3.7085	-0.0705	101	-0.8182	0.0041
30	-6.3770	-0.0819	66	-3.6168	-0.0693	102	-0.7655	0.0053
31	-6.3215	-0.0813	67	-3.5264	-0.0679	103	-0.7152	0.0064
32	-6.2663	-0.0818	68	-3.4324	-0.0654	104	-0.6673	0.0075
33	-6.2193	-0.0818	69	-3.3424	-0.0629	105	-0.6219	0.0084
34	-6.1478	-0.0772	70	-3.2536	-0.0600	106	-0.5788	0.0093
35	-6.0865	-0.0771	71	-3.1596	-0.0575	107	-0.5381	0.0101
						108	-0.4997	0.0108
Frobenius Norm $1 - \frac{\|\|e'e\|\|_F}{\|\|y'y\|\|_F} = 0.7398$								

Note: Author's calculus.

Fifth, age-specific constants $\mathbf{a}_x$ and $\mathbf{b}_x$ for both fertility and mortality were matched to period e_0 and TFR used in Lee et al. (2000, 2001, 2003). Sixth, I calculated the life table variables $\{l_{t,x}, p_{t,x}, q_{t,x}, f_{t,x} = \mathbf{f}(t,x)f_{fab}\}$, where f_{fab} is the fraction of female at birth, for $x \in \{0, \cdots, \Omega\}$ and $t \in \{1721, \cdots, 2250\}$, based on both the age-specific constants, obtained in step fifth, and demographic assumptions made for data before the demographic transition (1900) and after the demographic transition (2050), which are reported in Table 6.

Table 6: Characteristics of the Population Before and After the Demographic Transition

Characteristics	**Pre-Transition**	**Post-Transition**
Population growth rate (per year)	1.1%	0.0%
Life expectancy at birth (years)	28.3	78.8
Period-total fertility rate (TFR)	6.0	2.0
Age of retirement	65	65
Age of entrance into the labor market	21	21

Source: see Lee et al. (2000, 2001, 2003).

Seventh, I used projection matrices (see Equation 14) to obtain the population of pseudo-Taiwan from 1721 up to 2250, starting with a stable population given by a Lotka's r equal to 1.1%.

$$\begin{pmatrix} N_{t+1,0} \\ N_{t+1,1} \\ \vdots \\ N_{t+1,x} \\ N_{t+1,x+1} \\ \vdots \\ N_{t+1,\Omega-1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & f_{t,x} & f_{t,x+1} & \dots & 0 \\ \frac{l_{t+1,1}}{l_{t,0}} & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \frac{l_{t+1,x+1}}{l_{t,x}} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix} \cdot \begin{pmatrix} N_{t,0} \\ N_{t,1} \\ \vdots \\ N_{t,x} \\ N_{t,x+1} \\ \vdots \\ N_{t,\Omega-1} \end{pmatrix}, \quad (14)$$

Other economic assumption that affects on the demography is that individuals are not independent up to the age of 21. Thereby, I assume that individuals cannot give birth unless they are indepedent. As a consequence, it is necessary to readjust the fertility rates so as to keep the number of newborns at any given year unchanged (with respect to the original population projection). To do this I spread out for each year, the probability of giving birth from the age range 13-20 equally among the remaining fertile individuals; that is, individuals from age 21 to age 51.

$$\sum_{x=13}^{20} N_{t,x} f_{t,x} = c_t \sum_{x=21}^{51} N_{t,x} \Rightarrow c_t = \frac{\sum_{x=13}^{20} N_{t,x} f_{t,x}}{\sum_{x=21}^{51} N_{t,x}}. \quad (15)$$

Then, the age-specific fertility rates used in the simulations are:

$$\hat{f}_{t,x} = f_{t,x} + c_t, \forall t \in \{1721 \cdots 2250\}. \quad (16)$$

Ninth, I derived both the number of equivalent-adult-consumers that a household head of age x supports in year t, or $\lambda_{t,x}$, and the number of adult-surviving offspring in year t of an adult of age x, which is denoted by $o_{t,x}$. The mathematical expressions are:

$$\lambda_{t,x} = 1 + \sum_{z=T_w}^{x} \theta_{x-z} \frac{l_{t-x+z,z}}{l_{t,x}} l_{t,x-z} \hat{f}_{t-x+z,z} \cdot I_{x-z<T_w} \quad (17)$$

$$o_{t,x} = \sum_{z=\max\{T_w, x-T_r\}}^{x-T_w} \frac{N_{t-x+z,z}}{N_{t,x}} l_{t,x-z} \hat{f}_{t-x+z,z}, \text{ for } x > 2T_w, \quad (18)$$

where $I_{a<b}$ is a piecewise function that takes the value 1 when $a < b$ and 0 otherwise.

[Figure 8 about here.]

[Figure 9 about here.]

C Age-Specific Labor Productivity Indexes

In this model economy, I have assumed that all individuals have the same stock of human capital regardless when they are born, and thus age-specic labor productivity indexes do not change among generations. Also, by pooling in the demographic projections men and women, I am indirectly assuming that the labor force participation rates of both sexes are equal. In order to diminish this measurement error I use "Total Labor Earnings" by age supplied by the National Transfer Account (NTA) database for the period 1978 to 2003. These Total Labor Earnings are weighted each year by the size of each age-cohort. Consequently, I reduce this error because those cohorts with lower labor force participation rate will have less labor income per capita.

Since the simulated outputs are in real terms, I calculate total real labor earnings using the price index data from Taiwan[13]. Then, given that I have assumed as elsewhere that the structure of the salary has a multiplicative relation between the time component and the age component, I use the *singular value decomposition* method to calculate age-specific labor productivity indexes.

[13]See http://investintaiwan.nat.gov.tw/en/env/stats/gdp.html|

Proof. Let $\mathbf{Y} = [y_{t,x}]_{t=1978\ldots2003; x=T_w\ldots T_r-1}$ be a matrix $T \times N$ of salaries by age and time. Calling Equation (5) each entry can be divided into a time component, $\eta_t = w_t A_t$, and an age component, ϵ_x, as follows

$$y_{t,x} = w_t A_t \epsilon_x \Rightarrow y_{t,x} = \eta_t \epsilon_x.$$

Let define $\eta = [\eta_1, \eta_2, \ldots, \eta_T]'$ the vector of time components and $\mathbf{e} = [\epsilon_1, \epsilon_2, \ldots, \epsilon_N]'$ the vector of age component. Then, $\mathbf{Y}$ can be rewritten as

$$\mathbf{Y} = \eta \cdot \mathbf{e}' = \begin{pmatrix} \eta_1\epsilon_1 & \eta_1\epsilon_2 & \cdots & \eta_1\epsilon_N \\ \eta_2\epsilon_1 & \eta_2\epsilon_2 & \cdots & \eta_2\epsilon_N \\ \vdots & \vdots & \ddots & \vdots \\ \eta_T\epsilon_1 & \eta_T\epsilon_2 & \cdots & \eta_T\epsilon_N \end{pmatrix}. \quad (19)$$

Let the singular value decomposition of the matrix of salaries be

$$\mathbf{Y}_{T\times N} = \mathbf{U}_{T\times T} \cdot \mathbf{\Lambda}_{T\times N} \cdot \mathbf{V}_{N\times N}. \quad (20)$$

where $\mathbf{U}$ and $\mathbf{V}$ are basis of eigenvectors associated to matrices $\mathbf{YY'}$ and $\mathbf{Y'Y}$ respectively. Using (19) and (20) we obtain the following equalities:

$$\mathbf{YY'}_{T\times T} = \eta \cdot \mathbf{e}'\mathbf{e} \cdot \eta' = \mathbf{U}(\mathbf{\Lambda\Lambda'})\mathbf{U}'$$

and

$$\mathbf{Y'Y}_{N\times N} = \mathbf{e} \cdot \eta'\eta \cdot \mathbf{e}' = \mathbf{V}'(\mathbf{\Lambda'\Lambda})\mathbf{V}.$$

However, by definition there is only one non-zero eigenvalue equal to one, that is $[\mathbf{\Lambda}]_{1,1} = 1$ and $[\mathbf{\Lambda}]_{t,x} = 0, \forall t \neq 1$ and $x \neq 1$. Therefore, $\mathbf{Y}$ can be decomposed as

$$\mathbf{Y}_{T\times N} = \mathbf{u}_{T\times 1} \cdot \mathbf{v}'_{N\times 1} = \left(\frac{\eta}{\sqrt{\mathbf{e'e}}}\right)_{T\times 1} \cdot \left(\frac{\mathbf{e}}{\sqrt{\mathbf{f'f}}}\right)'_{N\times 1} \quad (21)$$

Finally, as we are just interested in relative prices, the fact that $\mathbf{e}$ is weighted by $\sqrt{\eta'\eta}$ does not affect to our problem. Figure 9 below reports the age-specific labor productivity index by age obtained using the singular value decomposition. The leading eigenvalue accounts for 79% of the total variability.

[Figure 10 about here.]

D Computational Details

For simplicity in the notation I remove the time indexes when they are not absolutely necessary, I denote with "′" the next period, and I will use the notation $\mathbb{R}_+$ to represent $\mathbb{R}^+ \cup \{0\}$.

D.1 Household Problem

The aim of the household head of age $x \in \mathcal{X} = \{T_w, \ldots, \Omega\}$ in year $t \in \mathcal{T} = \{t_0, t_0+1, \ldots, T\}$ is to maximize her expected utility by choosing the optimal consumption and assets in period $t+1$. The Bellman equation for the head of the household reads as

$$v(t, x, a|A) = \max_{c,a'} \left\{ \lambda_{t,x} u(c) + \beta p_{t,x} v(t+1, x+1, a'|A') \right\} \tag{22}$$

subject to

$$a' = \begin{cases} (1+r)a + h + (1-\tau^{oas})wA\epsilon_x - \lambda_{t,x}c & T_w \leq x < T_r \\ (1+r)a + h + \pi^{oas}wA\tilde{\epsilon}_x - \lambda_{t,x}c & T_r \leq x < \Omega \end{cases} \tag{23}$$

$$c, a \geq 0, \text{ with } a_{\cdot,T_w} = a_{\cdot,\Omega} = 0.$$

where $\beta \in (0,1)$ is the subjective discount factor, $p_{t,x} \in [0,1)$ is the probability of surviving to age $x+1$ in year $t+1$ conditional on being alive at age x in year t, h is the unintentional bequest received, τ^{oas} is the proportion of the salary spent to support her old parent, ϵ is the age-specific labor productivity index, π^{oas} is the proportion of the average labor income of her adult offspring for old-age support, A is the labor-augmenting technological progress, and a denotes asset holdings.

I define $G(a, a'|I)$ as the function of total amount of consumption good obtainable for any combination of assets held at any t and $t+1$, given the information I at t which depends upon the age of the individual and other time dependent variables $\{A, r, w, \lambda, \epsilon, \tilde{\epsilon}, h, \tau^{oas}, \pi^{oas}\} \in \mathbb{I}$, then $\mathbb{I} \subset \mathcal{X} \times \mathbb{R}^{+4} \times \mathbb{R}^5_+$. That is,

$$c = G(a, a'|I) = \begin{cases} \frac{1}{\lambda}\left((1+r)a - a' + h + (1-\tau^{oas})wA\epsilon_x\right) & T_w \leq x < T_r \\ \frac{1}{\lambda}\left((1+r)a - a' + h + \pi^{oas}wA\tilde{\epsilon}_x\right) & T_r \leq x < \Omega \end{cases}. \tag{24}$$

Let define the set $C \subset \mathbb{R}_+ \times \mathbb{R}_+$ the region of pairs $(a, a') \in \mathbb{R}^2_+$ where consumption is nonnegative; that is, $C = \{(a, a') \in \mathbb{R}^2_+ : G(a, a'|I) \geq 0, \text{ for any given } I\}$. It is easy to prove that C is a convex set. Now, using (24) let rewrite the Bellman equation as

$$v(t, x, a|A) = \max_{a'} \left\{ \lambda_{t,x} u(G(a, a'|I)) + \beta p_{t,x} v(t+1, x+1, a'|A') \right\}. \tag{25}$$

The algorithm operates on (25) for all individuals in each year up to the model converges. The algorithm involves the following steps:

1. Define a time-independent grid for assets, with $||a_{i+1} - a_i||$ sufficiently small,
$$G^a = \{a_1 = 0, a_2, a_3, \ldots, a_n\}, \tag{26}$$
where a_n is the maximum realization of assets weighted by units of effective labor.

2. Define the correspondence $f : G^a \rightarrow G^a$ of optimal combinations of assets in t and $t+1$ at age x.
$$f(a_k|I, A') = a_j^* = \arg\max_{a_j} \left\{\lambda_{t,x} u(G(a_k, a_j|I)) + \beta p_{t,x} v(t+1, x+1, a_j|A')\right\}, \tag{27}$$
where $I \in \mathbb{I}$ and $A' \in \mathbb{R}^+$ is the productivity in year $t+1$. Note that f is a one-to-one correspondence given that C is a convex set and $u(\cdot)$ is strictly concave.

3. Calculate the set $\{(a_k, f(a_k|I, A'))\}_{k=1}^n \in C^{2n}$ of all possible optimal asset pairs for the household head in year t. Evaluate Equation (25) by introducing all optimal asset pairs. Repeat this process all through the life cycle of the household head.

4. Repeat step 3 for all households.

5. Given the initial boundary conditions we know that wealth at the beginning of adulthood is zero, or $a_{t,T_w} = a_1 \in G^a$, $\forall t \in \mathcal{T}$. Therefore, given all information sets over the life cycle of the household head and Equation (27) I iterate forward on age and time to get the optimal path of asset holdings.

6. Finally, repeat step 5 for all individuals.

D.2 Aggregate Model

In this model the equilibrium price vector is numerically obtained using the Gauss-Seidel algorithm, see Auerbach and Kotlikoff (1987) and Börsch-Supan et al. (2006) among others.

The simulation strategy was to calculate the demand and supply of capital at all times for a given vector of interest rates $\{r_t^i\}_{t=0}^T$, with T sufficiently large and i denoting the i-th iteration, such that there is no excess of demand of capital at any time. The information set prior to the simulation is a vector of time-invariant parameters and demographic characteristics for $t \in \mathcal{T}$. In order to guarantee the existence of an equilibrium, the phase-in (out) period begins (finishes) with a stable population 200 years before (after) the period being analyzed, so that the economy before and after the demographic transition is in a steady-state equilibrium. The algorithm is divided into the following seven steps:

1. Choose a dumping factor of $\xi = 0.05$ and a tolerance ϵ equal to 0.02.

2. Choose an initial guess $\{R_t^0\}_{t=0}^T$, where R_t^i is equal to $r_t^i + \delta$ for all $i \in \mathbb{N}$, in which the initial and final steady-state interest rates are included.

3. Given the initial guess, calculate using a Cobb-Douglass production function its associated salary over time in units of effective labor

$$w_t^i = (1-\alpha)\left(\frac{R_t^i}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \forall t \in \mathcal{T}. \tag{28}$$

4. Compute the household problem and aggregate assets across all household heads to determine the capital stock per units of effective labor

$$\kappa_t = \frac{\sum_{x=T_w}^{\Omega-1} a_{t,x} N_{t,x}}{\sum_{x=T_w}^{T_r-1} A_t h_x N_{t,x}}, \forall t \in \mathcal{T}, \tag{29}$$

5. Next, determine the marginal product of capital resulting from (29), that is

$$r_t^n + \delta = \alpha \kappa_t^{\alpha-1}, \forall t \in \mathcal{T}. \tag{30}$$

6. If $||\mathbf{r}^i - \mathbf{r}^n|| < \epsilon$ then STOP.

7. Else, compute a new vector of interest rates and salaries

$$r_t^{i+1} = (1-\xi) r_t^i + \xi r_t^n, \tag{31}$$

$$w_t^{i+1} = (1-\alpha)\left(\frac{r_t^{i+1}+\delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \forall t \in \mathcal{T}. \tag{32}$$

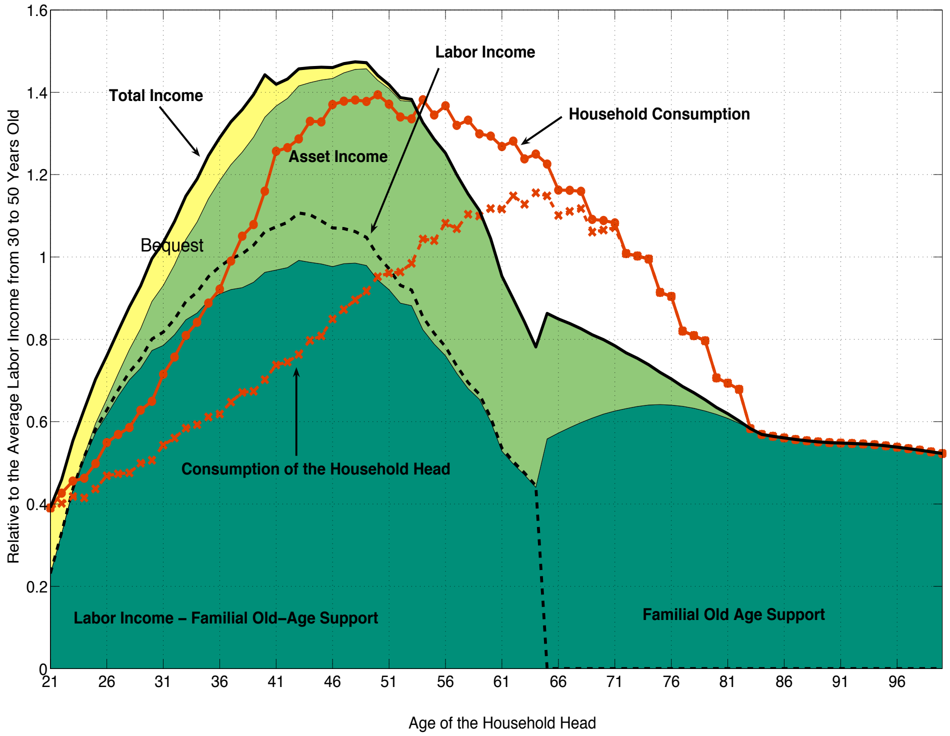
8. Calculate the unexpected bequest

$$\begin{aligned} h_{t,x} &= (1+r_t^{i+1}) \sum_{s=T_w}^{x} \frac{N_{t-x,s}\hat{f}_{t-x,s} f_{fab}}{N_{t-x,0}} \frac{N_{t,s+x} d_{t,s+x}}{o_{t,s+x}} a_{t,s+x} \\ &+ (1+r_t^{i+1}) \frac{q_{t,x}}{p_{t,x}} a_{t,x} I_{x<2T_w}, \end{aligned} \tag{33}$$

for all $x \in \mathcal{X} = \{T_w, \ldots, \Omega\}$ and $t \in \mathcal{T} = \{t_0, t_0+1, \ldots, T\}$. Then go to step 4.

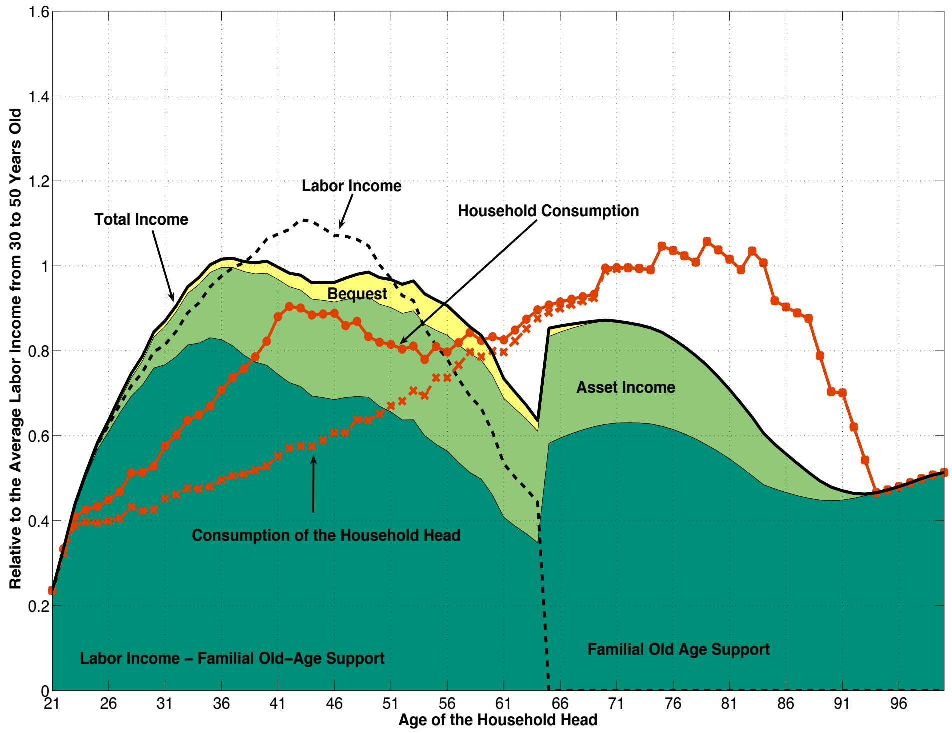
List of Figures

Figure 1: Pseudo-Taiwan. Simulated Income, Consumption, Transfer, and Saving Profiles for a Representative Individual before the Demographic Transition.



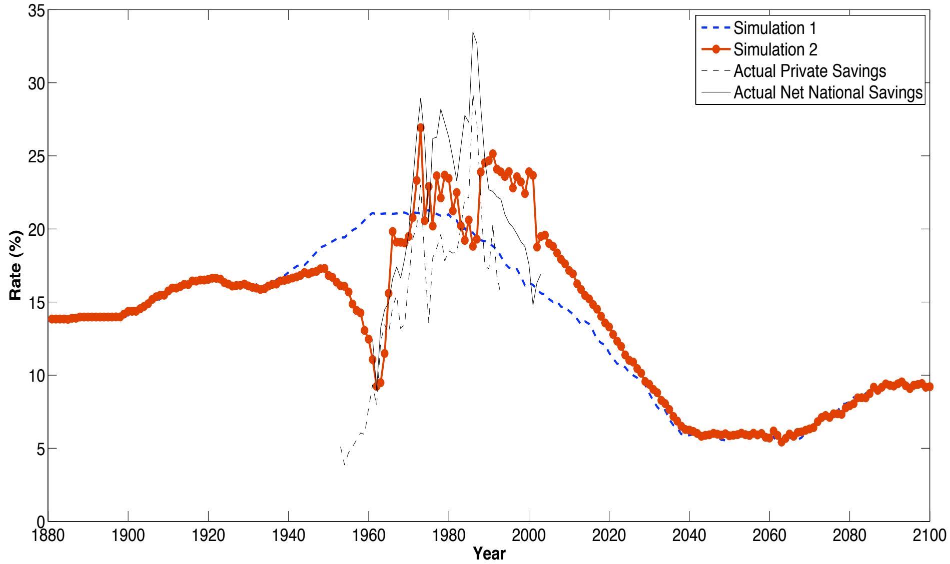
NOTES: All income sources are plotted in stacked format. All age profiles are standardized longitudinally using the average labor income from 30 to 50 years old of the same cohort. The difference between labor income (black dotted line) and labor income minus Familial old-age support (dark green shadow before age 65) is the cost for the individual to financially support her elder parents.

Figure 2: Pseudo-Taiwan. Simulated Income, Consumption, Transfer, and Saving Profiles for a Representative Individual after the Demographic Transition.



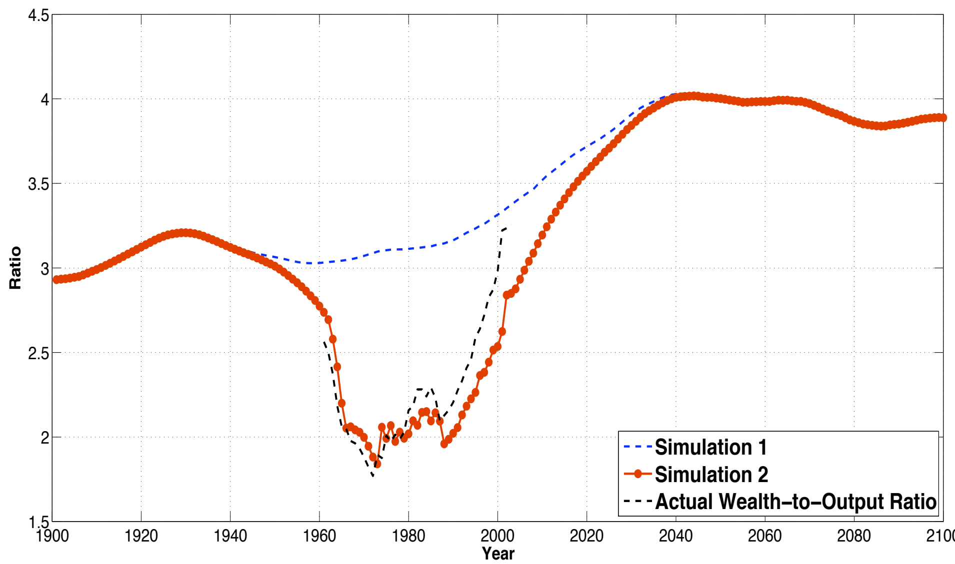
NOTES: All income sources are plotted in stacked format. All age profiles are standardized longitudinally using the average labor income from 30 to 50 years old of the same cohort. The difference between labor income (black dotted line) and labor income minus Familial old-age support (dark green shadow before age 65) is the cost for the individual to financially support her elder parents.

Figure 3: Pseudo-Taiwan. Simulated Saving Rates, 1880-2100.



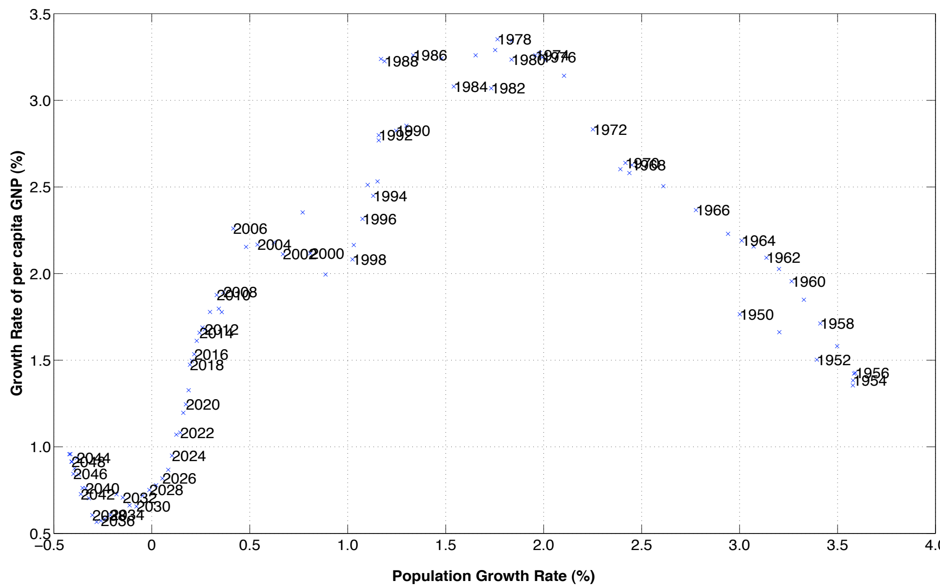
NOTES: In Simulation 1 the labor-augmenting technological progress is 1.5 percent. Simulation 2 the the labor-augmenting technological progress equals 1.5 percent pre-1962 and post-2003, from 1962 to 2003 is displayed in Figure 7.

Figure 4: Pseudo-Taiwan. Wealth-to-Output Ratio, 1900-2100.

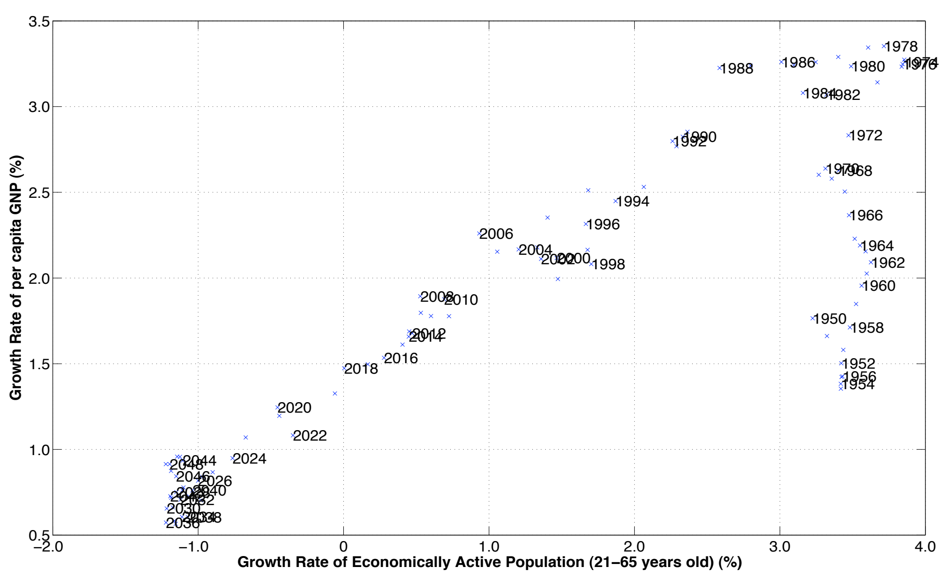


NOTES: In Simulation 1 the labor-augmenting technological progress is 1.5 percent, whereas in Simulation 2 the labor-augmenting technological progress equals 1.5 percent pre-1950 and post-2000, from 1962 to 2003 is displayed in Figure 7.

Figure 5: Pseudo-Taiwan. Growth Rate of GDP per capita from 1950 to 2050 under Simulation 1.

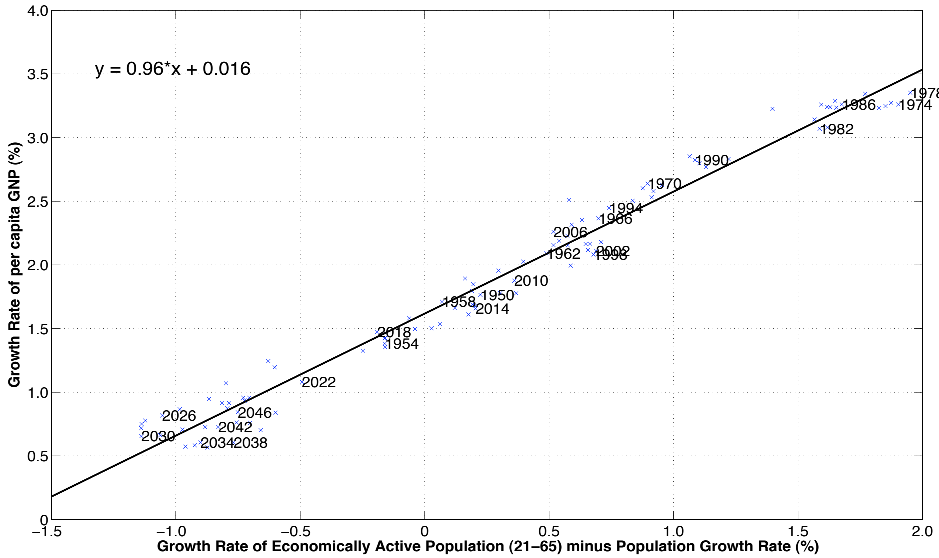


(a) Growth Rate of GDP per capita versus Population Growth Rate.

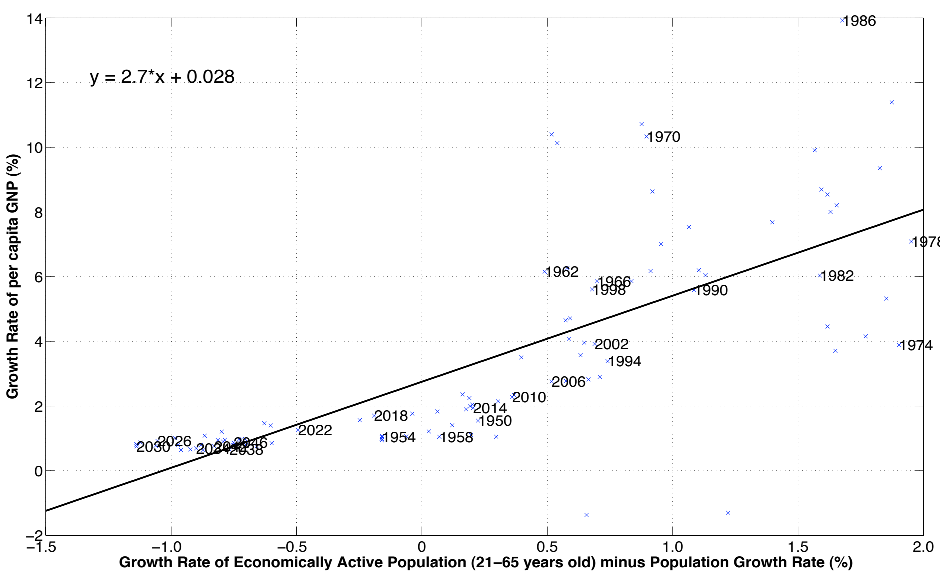


(b) Growth Rate of GDP per capita versus Growth Rate of Economically Active Population.

Figure 6: Pseudo-Taiwan. Growth Rate of GDP per capita versus grL-grN, 1950-2050.

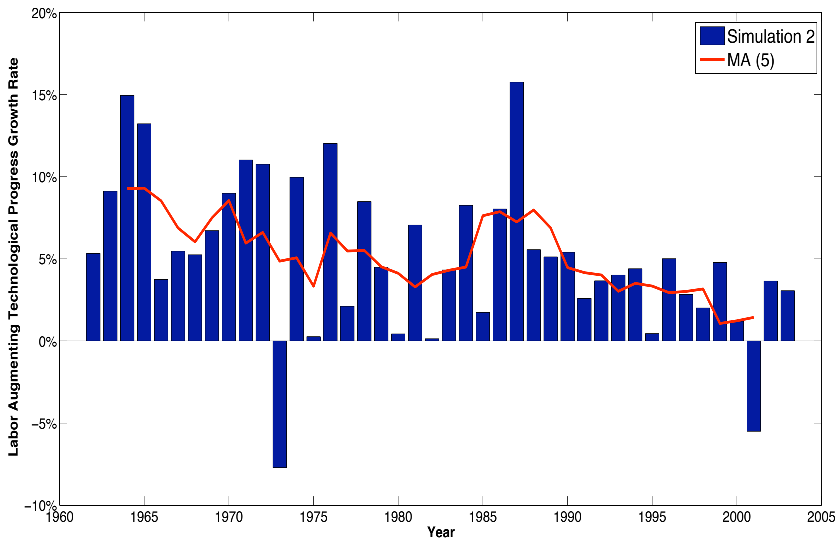


(a) Simulation 1. Constant Labor-Augmenting Technological Progress (A) 1.5%



(b) Simulation 2. Labor-Augmenting Technological Progress equals 1.5% pre-1950 and post-2000, from 1962 to 2003 A is displayed in Figure 7.

Figure 7: Taiwan. Estimated Labor Augmenting Technological Progress from 1962 to 2004.



NOTES: Red solid line is the associated moving average of order 5.

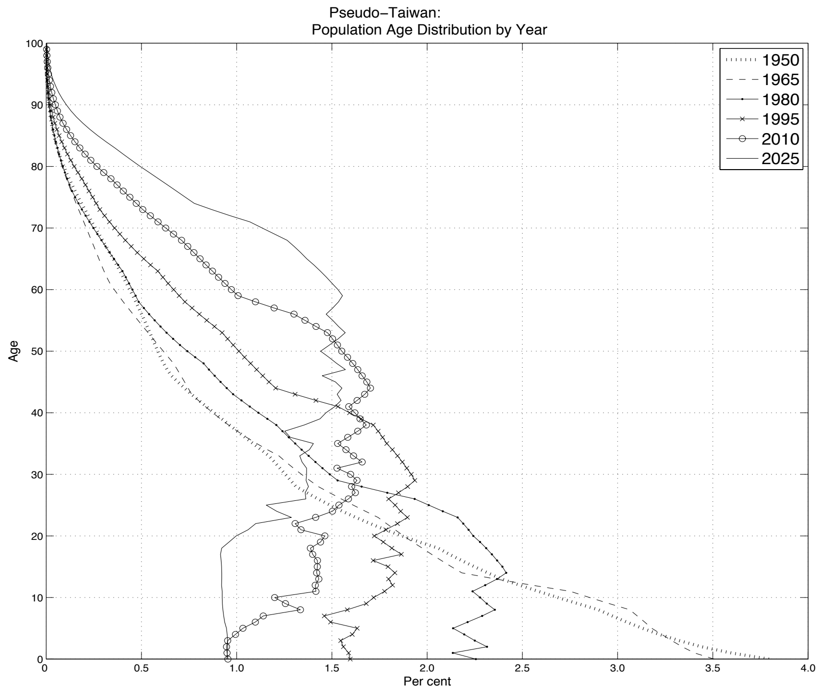

Pseudo−Taiwan:
Population Age Distribution by Year
Age
Per cent
1950
1965
1980
1995
2010
2025
0
10
20
30
40
50
60
70
80
90
100
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Figure 8: Pseudo-Taiwan: Projected Dependency Ratios from 1950 to 2050.

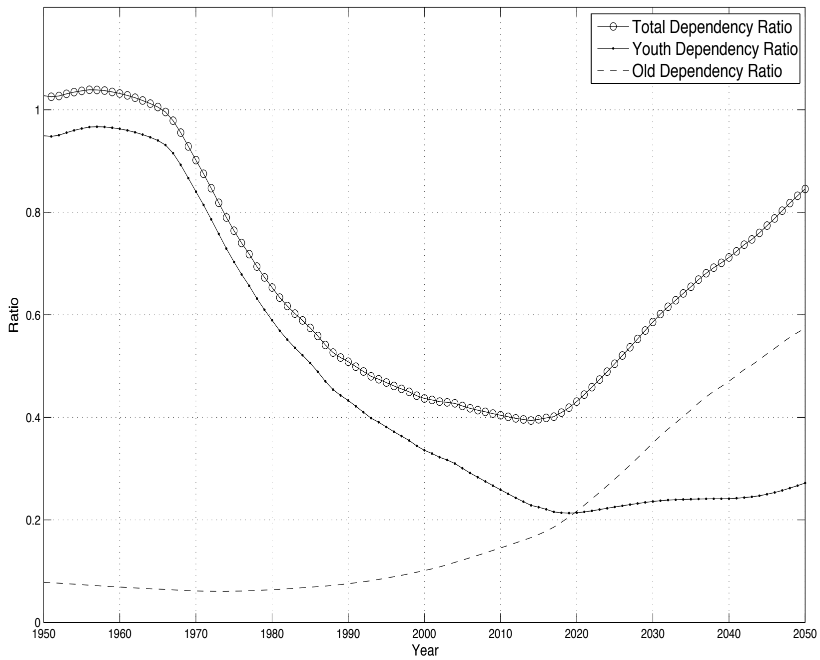


Figure 9: Pseudo-Taiwan: Estimated Age-Specific Labor Productivity Index